Analysis and Comparison of FEC and FEC-ARQ Protection Schemes based on RS and Raptor Code

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Abstract—**Four protection schemes based on RS and Raptor code as well as FEC and hybrid FEC-ARQ are analyzed and compared for erasure channels in this paper. RS code is a classic block channel code while Raptor code is a new kind of rateless code, which can produce a potentially infinite stream of packets. FEC and hybrid FEC-ARQ schemes based on RS and Raptor code are described and compared in detail. Then numeric analysis and experiments are conducted on these schemes in terms of several performance measures. The results demonstrate that Raptor code based schemes can effectively reduce coding complexity and FEC-ARQ protection mechanism can effectively increase channel utility.**

Keywords- RS code, Raptor code, FEC, hybrid FEC-ARQ

I. INTRODUCTION

In the network transmission, forward error correction (FEC) is the most common sender-based loss recovery technique for data protection. Traditional FEC schemes usually select a simplex channel model for transmission: the sender utilizes an erasure channel code, e.g. Reed-Solomon (RS) Code, to encode the source packets while the receiver performs decoding accordingly. There has been a lot of work focusing on designing and optimizing these schemes. Shan et al. [1] proposed a two-stage FEC scheme using RS code and interlacement. Bolot et al. [2] worked on suitable FEC schemes for Internet telephony. However, several problems exit in such schemes:

- In the FEC scheme, decoding fails in case of insufficient packets, leading to a serious waste of bandwidth as the available packets cannot be further utilized.
- The FEC scheme usually cannot adapt to channels of abruptly high loss rate.

Automatic Repeat Request (ARQ) is another common receiver-based loss recovery technique, which is combined with FEC in a lot of work to alleviate the above problems. A FEC-ARQ protocol that required a specially designed "streaming code" was described in [3]. The sender in [4] sent the whole code block again to respond to the ARQ. And in [5] the advantage of RS based FEC-ARQ strategy was also described over the meteor burst channel. But with the traditional block codes, there still exist some problems in hybrid schemes as follows.

- The sender has to reserve encoded packets for possible retransmission, which brings extra memory consumption.
- The receiver's feedback has to indicate the necessary retransmission information, which requires additional protection consumption as well.

The Raptor code [6] [7], as a kind of fountain code [8], performs well in packet loss recovery and has received increasing popularity in recent years. For example, it is used to provide protection for video transmission in [9~11]. In contrast to block codes, Raptor code can theoretically produce a length-infinite stream of packets, which enables it to be rateless and adapt well to the FEC-ARQ mechanism.

There has been a lot of work comparing various FEC schemes. FEC based on RS code and Raptor code was analyzed in [12]. In [13] FEC was compared to MDC techniques for Voice over IP. Besides, much work [4, 14] focused on the application of FEC-ARQ based on traditional block codes. However, the analysis and comparison of Raptor and RS code based FEC and FEC-ARQ schemes is still desired, as no sufficient demonstration is presented for the Raptor code applied in FEC-ARQ schemes.

In this paper, as two representative channel codes, the RS code and the Raptor code are combined with FEC and hybrid FEC-ARQ mechanisms to produce four protection schemes, which are analyzed and compared. Several performance measures are also defined for the purpose of evaluation. Then we conduct the analysis and comparison via numeric experiments and simulation. The weakness and strength of the four schemes are demonstrated in the final results.

The rest of the paper is organized as follows. In Section 2, the features of representative channel codes are described. Relevant transmission schemes are also presented. In Section 3, these schemes are analyzed and compared in detail. Numeric experiments showing the performance of these schemes are given in Section 4. And Section 5 draws conclusions.

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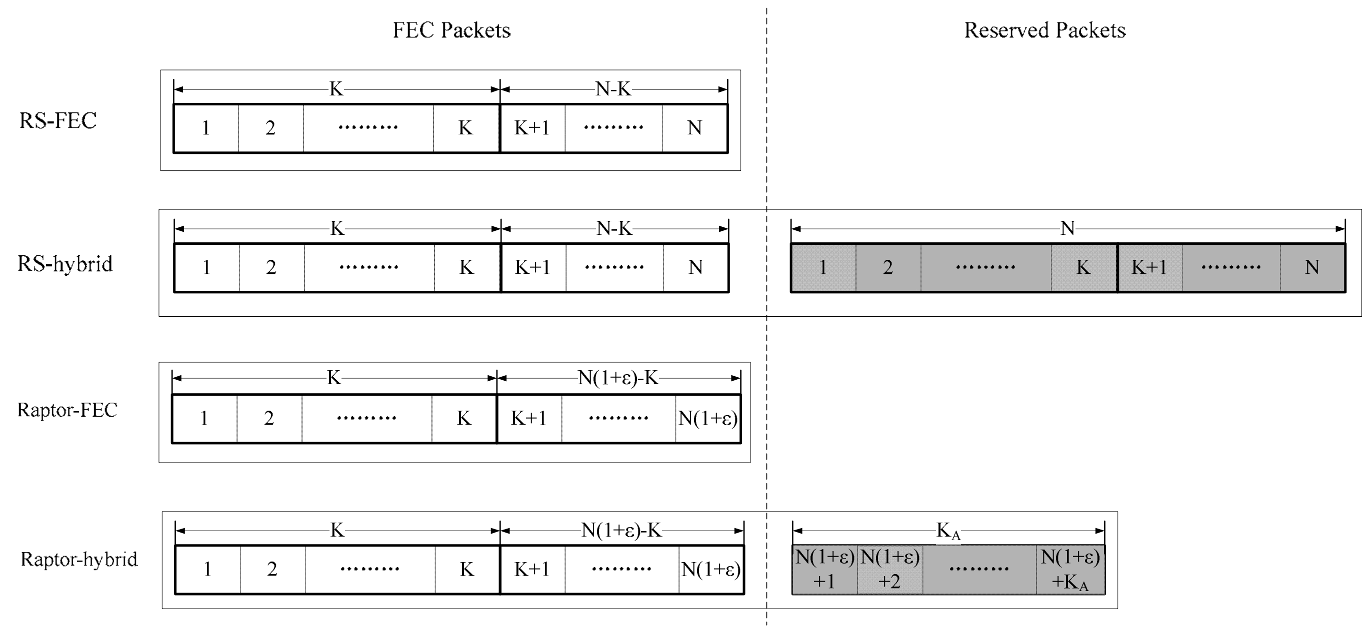

Figure 1. Illustration of the four protection schemes

II. REPRESENTATIVE CHANNEL PROTECTION SCHEMES

In this paper, RS code and Raptor code, as a representative block code and rateless code respectively, are combined with FEC and hybrid FEC-ARQ mechanisms to produce four protection schemes: RS-FEC, RS-hybrid, Raptor-FEC and Raptor-hybrid. And these schemes are introduced on aspect of their channels codes and loss recovery mechanisms in this section.

A. Features of RS Code and Raptor Code

In this part, the features of RS code and Raptor code are presented for erasure channels under the assumption that lost packets can be exactly located at the receiver side.

The RS code is a kind of maximum distance separable (MDS) code, whose coding theory is based on Galois Field $\left(2^m\right)$. Let N and K denote the code length and information length respectively. The codeword can be described by RS (N,K) and has a length-fixed block code structure with coding complexity $O\big(K(N-K)\log_2 N\big)$. RS code can ensure correct recovery as long as K out of the N packets are received.

The Raptor code [6] is a class of rateless code with a computational complexity $O(N\log_2 K)$ if N packets are produced from K source packets. As a rateless code, Raptor code can theoretically produce a length-infinite stream of encoded packets named "fountain" packets. Raptor code has the overhead ε that affects the possibility of successful decoding p_ε, which is close to 1 if more than $K(1+\varepsilon)$ packets are received and unsure otherwise. Experimental results show that when K is 300, 500, 1000, and 2000 respectively, ε equals to 0.01, 0.006, 0.005, and 0.0055 respectively. And when K is set equal to 1000, if 1000, 1001, 1002, 1003 or 1004 packets are received, p_ε equals to 0.26, 0.37, 0.71, 0.79, or 0.78 respectively.

B. FEC and hybrid FEC-ARQ

Prior to transmission across the network, FEC provides reliable redundant packets to be appended to the source packets, which can be implemented by suitable channel codes. In case some packets get lost during transmission, FEC enables the receiver to use the additional packets to decode and recover the lost ones.

Hybrid FEC-ARQ mechanism provides extra packets to be repeated once FEC fails, when the receiver sends an acknowledge signal (.i.e. ACK/NAK) and feedback information of necessary loss location to the sender.

In the RS hybrid scheme, during retransmission the sender has to repeat the whole block of packets, unless the feedback information also arrives to direct the sender to repeat only the designated packets. In this paper the RS hybrid scheme is analyzed with the feedback location information assumed to be available.

In contrast, in the Raptor-hybrid scheme, no feedback location information is required during retransmission, as any backup packets contribute to successful decoding.

The packetization principles of the four schemes are illustrated in Figure 1. Both hybrid schemes reserve packets to prepare for retransmission. Raptor based schemes are designed with a code length of $N(1+\varepsilon)$ rather than N, as the Raptor code is rateless and the possibility of successful decoding is high only if more than $K(1+\varepsilon)$ packets are received.

III. Analysis and Comparison

In this section, the performance of RS-FEC, RS-hybrid, Raptor-FEC and Raptor-hybrid channel protection schemes is analyzed and compared.

Assume N packets are produced from K source packets for all schemes, and extra packets are only produced on backup purpose for the Raptor-hybrid scheme. Given the channel packet loss rate ρ, the channel code rate r (the ratio of information length K to code length N) is assume to be the critical value $(1-\rho)$ and N hence equals to $K/(1-\rho)$. This critical rate implies the number of received packets at the receiver side is expected to be K.

A. Computational Complexity

Computational complexity, highly dependent on the channel code applied in a scheme, is the major measure of the encoding and decoding speed of the scheme.

As mentioned in Section 2.1, RS based schemes require the coding complexity of $O\left(K(N-K)\log_2 N\right)$ to code N packets. This implies its limitation that they are practical for small N and K. By contrast, Raptor based schemes have the coding complexity of $O(N)$ to code N packets, namely a linear complexity, which is much lower than that of RS based schemes. Therefore large block of packets can be coded in a short period and channel code of long length is practical.

B. Extra Memory Consumption

In the hybrid schemes, decent amount of packets are reserved at the sender side in the FEC-ARQ mechanism, leading to extra memory consumption. Let E denote the number of packets reserved and c_m denote this consumption rate. Then c_m can be calculated as

$$c_m = E/N. \tag{1}$$

Due to the fixed block structure of RS codes, the RS sender has to reserve all the packets in a code block as any one may be lost. That's to say RS schemes have an extra memory consumption rate of 1.

In contrast, Raptor codes have the advantage that any subset of backup packets contributes to decoding. In other words, it's no longer necessary to reserve all the N packets but only the backup packets. Let K_A be the number of repeated packets. We can get the consumption rate for the Raptor-hybrid scheme

$$c_m = K_A/N. \tag{2}$$

which is usually far less than 1.

It's obvious that c_m is 0 in FEC schemes.

C. Extra Protection Consumption

Feedback information is transmitted in hybrid schemes, which require extra protection consumption compared to FEC schemes.

We use the feedback information to denote the extra protection consumption. According to Section 2.2, ACK/NAK signal is the common protection consumption in both hybrid schemes but feedback location information is necessary for the RS-hybrid scheme only.

D. Utility of Channel Bandwidth

Suppose data is transmitted over a channel of bandwidth B. Let r denote the channel code rate, which is the ratio of information length to code length of channel code. So the effective bandwidth at the sender side is $B\cdot r$. Given the possibility of successful channel decoding p_s, we may try to maximize the effective information throughput at the receiver side, which can be calculated as

$$V = B\cdot r\cdot p_S \tag{3}$$

Assume the channel bandwidth keeps constant. Then we can focus on the maximization of ultimate utility of channel bandwidth

$$U = r \cdot p_S. \tag{4}$$

The utilities of the four schemes can be formulated as follows.

In a stochastic packet loss channel, the probability that out i of N packets can be received is

$$p_r(i) = \binom{N}{i}\cdot(1-\rho)^i\cdot\rho^{N-i}. \tag{5}$$

If P_f is the probability that decoding fails, then RS decoding fails at the probability

$$P_f = \sum_{i=0}^{K} p_r(i), \tag{6}$$

and Raptor decoding fails at

$$P_f = \sum_{i}^{K(1+\varepsilon)} p_r(i) - \sum_{i=K}^{K(1+\varepsilon)-1} p_r(i)\cdot p_\varepsilon(i-K) \approx \sum_{i}^{K(1+\varepsilon)} p_r(i), \tag{7}$$

where P_ε is defined in Section 2.1. Then U can be formulated as follows.

- FEC Schemes

According to (4), we have the bandwidth utilities

$$U_{FEC} = r\cdot P_S = r\cdot(1-P_f). \tag{8}$$

- Hybrid Schemes

The hybrid schemes are analyzed under the assumption the acknowledge signal is always correctly received. ACK

TABLE I. COMPARISION OF THE FOUR SCHEMES ($K = 1000$, $\rho = 0.20$, $N{=}K/(1{-}\rho)$).

Schemes	Measures			
	Computational complexity	Extra memory consumption rate	Extra protection consumption	Bandwidth utility
RS-FEC	$O(K(N-K)\log_2 N)$	0	0	0.4135
RS-hybrid	$O(K(N-K)\log_2 N)$	1	Acknowledge signal, Loss location information	0.6530
Raptor-FEC	$O(N)$	0	0	0.4070
Raptor-hybrid	$O(N+K_A)$	K_A/N	Acknowledge signal	0.6117

happens at the probability p_A (equal to $1-P_f$) and NAK at p_N (equal to P_f). Then U_{hybrid} is computed as

$$U_{hybrid}=p_A U_A+p_N U_N . \tag{9}$$

The related parameters in (9) can be referred to in the Appendix.

IV. Experiments and Numeric Simulation

The number of repeated packets K_A is decided by the loss location information in the RS-hybrid scheme, but optimization of K_A is necessary in the Raptor-hybrid scheme according to (9) and (16). So the first experiment is carried out to find the optimized K_A for optimal bandwidth utility of Raptor-hybrid schemes under specific experiment conditions of fixed packet loss rate ρ and information length K :

Channel packet loss rate ρ : 0.20

Code rate r : 0.80

Codeword (N,K) : (1250, 1000)

Related measures are then calculated to show the weakness and strength of each scheme. The results in Figure 2 show that the maximum value of U_{hybrid} is 0.6117with optimal K_A of 50 for Raptor-hybrid.

After finding the optimal K_A for Raptor-hybrid, c_m is 0.04 according to (2), and utilities can also be calculated according to (8) and (9). Then, numerical comparison of the four schemes is shown in Table 1, from which it can be seen that:

- Raptor based schemes have much lower computational complexity than RS based schemes.
- Hybrid schemes require extra memory and protection consumption. It's worth noticing that the Raptor-hybrid scheme requires as little as 0.04 extra memory consumption rate and protection only for ACK/NAK. It's much lower than the RS-hybrid scheme, which has extra memory consumption rate of 1 and requires extra protection for loss location information.
- Both hybrid schemes have higher utilities of channel bandwidth than FEC schemes.

Now we focus on the bandwidth utilities of these schemes for different packet loss rate ρ and information length K . And the results are shown in Figure 3~5 for K = 300, 500 and 1000 respectively. It can be observed that all bandwidth utilities drop with the increase of packet loss rate ρ , as fewer packets can be received in case of higher ρ . Besides, hybrid schemes have higher utilities than FEC schemes, which can be attributed to the retransmission mechanism of the hybrid schemes. These packets required by the retransmission mechanism don't bring down the channel code rate a lot but increase the probability by a considerable extent.

What's more, the reason of RS-hybrid having the highest utility is that it obtains the loss location information from the receiver. But the RS-hybrid scheme owns much more extra protection consumption and higher coding complexity, which may make it less flexible for actual channel protection compared to the Raptor-hybrid scheme.

V. Conclusion

Four protection schemes are analyzed and compared, which are RS-FEC, RS-hybrid, Raptor-FEC and Raptor-hybrid. In contrast to schemes based on RS code, those based on Raptor code provide extremely low computational complexity, which makes them suitable for transmission of large block of data such as video. Hybrid FEC-ARQ schemes have relatively more memory and protection consumption as well as more complex operations, but the gain is higher utility of channel bandwidth compared to pure FEC schemes. It deserves notice that the Raptor-hybrid scheme doesn't bring much extra consumption but increases the bandwidth utility a lot. We conclude that the Raptor based hybrid FEC-ARQ scheme is more effective and flexible for data transmission.

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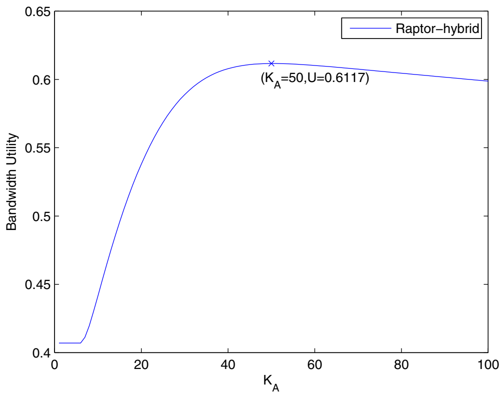

Figure 2. Bandwidth utility of Raptor-hybrid scheme with K=1000 and ρ=0.20

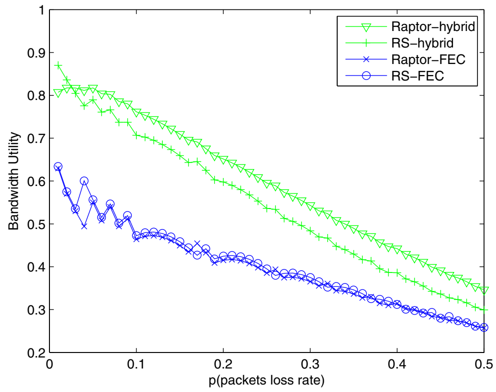

Figure 3. Bandwidth utility of the four schemes with K = 300 and ρ ranges from 0.01 to 0.50

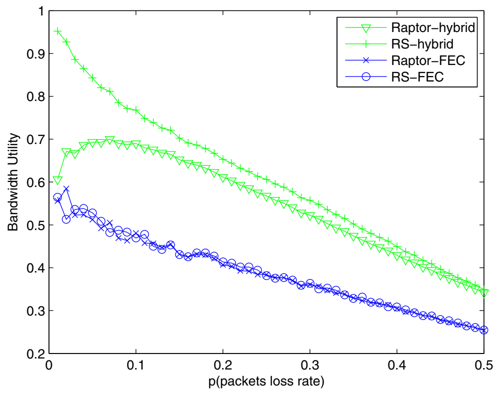

Figure 4. Bandwidth utility of the four schemes with K = 500 and ρ ranges from 0.01 to 0.50

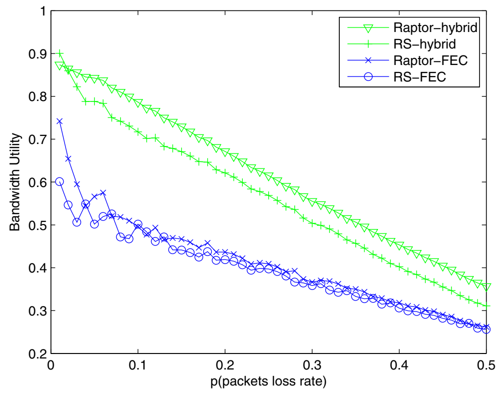

Figure 5. Bandwidth utility of the four schemes with K = 1000 and ρ ranges from 0.01 to 0.50

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APPENDIX

The parameters in (9) are deduced as below.

When ACK arrives, implying successful decoding and that $P(s|ACK)$ equals to 1, the sender continues sending the next code block. Note that the code rate r_A at this time is $(1-\rho)/(1+\varepsilon)$ for Raptor codes and $(1-\rho)$ for RS codes. So we have

$$U_A = r_A \,. \tag{10}$$

When NAK arrives, the sender repeats K_A packets. At this time the code rate r_N equals to $\frac{K}{(N+K_A)}$, and U_N can be expressed as

$$U_N = r_N \cdot P(s|NAK). \quad (11)$$

Now we analyze the probabilities of successful decoding after K_A packets are repeated for RS-hybrid and Raptor-hybrid separately.

For the Raptor-hybrid scheme, this probability is

$$P(s \mid NAK) = \sum_{i=0}^{K(1+\varepsilon)-1} p_{cr}(i) \times \left[1 - \sum_{j=0}^{\min(K_A, K(1+\varepsilon)-i)} \binom{K_A}{j} \times (1-\rho)^j \times \rho^{(K_A-j)}\right], \quad (12)$$

where $p_{cr}(i)$ is the probability that i packets have already been received when NAK is acknowledged. If $i \le K$, $p_{cr}(i)$ can be estimated as

$$p_{cr}(i) = \frac{p_r(i)}{p_f}, \quad (13)$$

and if $K(1+\varepsilon) > i \ge K$, it becomes

$$p_{cr}(i) = \frac{p_r(i) \cdot (1 - p_\varepsilon(i-K))}{P_f}. \quad (14)$$

For the RS-hybrid scheme, all the lost packets are repeated. So we have

$$P(s|NAK) = \sum_{i=0}^{K-1} \rho_{cr}(i) \cdot \left[1 - \sum_{j=0}^{K-i} \binom{K-i}{j} \cdot (1-\rho)^j \cdot \rho^{(K-i-j)}\right] \quad (15)$$

As p_A and U_A in (9) are independent of K_A, the optimization of bandwidth utility for the Raptor-hybrid scheme can be formulated as

$$\max_{K_A} U_N = \frac{K}{(N+K_A)} \cdot \left\{ \sum_{i=0}^{K(1+\varepsilon)-1} p_{cr}(i) \cdot \left[1 - \sum_{j=0}^{\min(K_A, K(1+\varepsilon)-i)} \binom{K_A}{j} \cdot (1-\rho)^j \cdot \rho^{(K_A-j)}\right] \right\}.$$

$$s.t. \quad K_A \ge 0$$

(16)