Interpolation Method Based Adaptive Directional Lifting Wavelet Transform

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Abstract—**This paper presents an interpolation method based on adaptive directional lifting wavelet construction. This approach can not only adaptively choose the best direction as lifting direction instead of horizontal and vertical direction, but also use the Lagrange's interpolation to predict pixels according to the local characteristics between pixels. Experimental result shows that the proposed method has advantages over the traditional lifting wavelet.**

***Keywords-** lifting wavele; adaptive direction; Lagrange's interpolation*

I. INTRODUCTION

Two-dimensional (2D) discrete wavelet transform (DWT) is the most important image compression technology in the past decade [1]. Typically, 2DWT is achieved by both horizontal and vertical directions one-dimensional wavelet transform. Thus wavelet transform only considers the horizontal and vertical directions in image compression, and can not effectively reflect the image orientation features, such as edge and lines. Therefore, wavelet structures based image edge and texture were studied by many researchers since emerging the wavelet transform. Lifting structure proposed by W.Sweldens is effective and popular. Lifting structure is more efficient by using causal connection of the neighbor pixels, and it is better to achieve clear geometric characteristics of the local prediction, because lifting structure must introduce neighbor pixels to predict and update pixels.

In order to achieve more accurate prediction structure, many scholars have done many research at space geometric properties of structure by finding more precise local boundary. And these structures are based on the direction wavelet. In Pier Luigi[3] paper, the authors proposed predict method of direction wavelet based on crystal lattice theory. This method applied to the local geometric properties of wavelet structure through a reasonable choice of the size of the lattice. Wenpeng Ding proposed an adaptive lifting wavelet structure[4],[5]. This method is not only more accurate to judge the best direction of wavelet lifting by the half, a quarter or even one-eighth of the interpolation and alteration of pixels, but also make the right judgments of the direction lifting wavelet by introducing quad-tree structure.

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Although no limited to horizontal and vertical directions, they neglected directional of horizontal lifting after vertical lifting in these mentioned directional wavelet structure. These do not consider the case of exited several border in local space. This paper will also judge the direction of vertical lifting in order to consider the appearance of local space of exceeding one border, especially the case of two strong borders.

Moreover, if using the original wavelet structure prediction, the pixel is not reflected in the relativity after choosing the best direction. In [6], the authors proposed a weighting algorithm according to different cases of predicting relativity of its neighbor pixels. This algorithm is only for its neighboring pixels to predict, but this paper considers details of the characteristics of every pixel in prediction. So that each pixel maybe has different lifting forms to reflect other local characteristics apart from directional property. We use interpolation prediction to achieve more accurate prediction according to the strong causality of the neighbor pixels.

II. ADAPTIVE DIRECTIONAL LIFTING WAVELET

In the images, different regions have different directions of the character of the geometric characteristics of space, such as border or stripe. If all of the geometric characteristics of this geometry deal with the lifting with the same structure, this may cause some ignored features or larger error. Therefore we focused on the geometric characteristics of the space relative to the most suitable direction of lifting.

We consider the adaptive direction no longer limited to horizontal and vertical directions. We select the set of direction as in Fig. 1. In the Fig. 1, white points represent odd samples and black points represent even samples. Here we only check for horizontal or vertical discrete direction between -45 degrees to 45 degrees, because we found that the absolute degrees which are greater than vertically direction degrees can be achieved by the absolute degrees which are less than horizontal direction. In this way, if there are two different direction characteristics in the image, we can use a direction parallel horizontal direction, then choose the other direction to adjust vertically direction.

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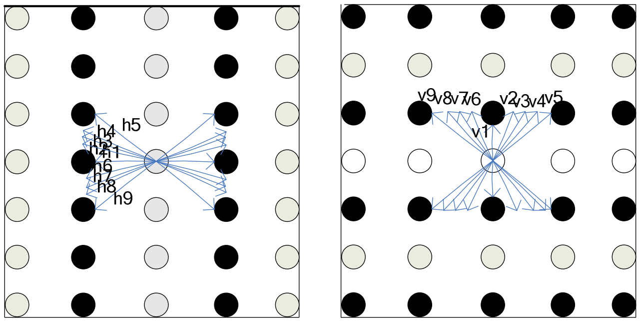

Figure 1. left image is horizontal lifting direction, right image is vertical lifting direction

Adding such a direction to select the structure in the basic structure of the lifting wavelet of split, predict and update steps. Since horizontal and vertical structures are similar lifting by 1-D signal expect different definitions of the direction, here we only consider the horizontal structure.

In order to maintain the original space direction of the image characteristics, we still use the traditional split of lifting wavelet. Two-dimensional signal $x(m,n)_{m,n\in z}$ will be split into even $x_e(m,n)=x(m,2n)$ and odd $x_o(m,n)=x(m,2n+1)$ set.

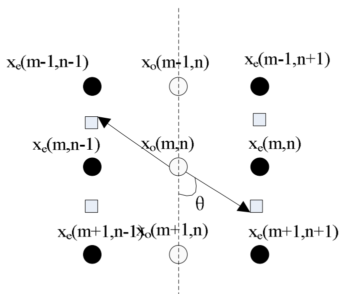

Figure 2. directional predict

The prediction process, in Fig. 2, assuming that the direction selected for the θ, and changing the original operator P into

$$P(x_e,m,n)=\sum_i a_i x(m+sign(i-1)\tan\theta,2n+i) \quad (2.1)$$

From the formula 2.1 we can see that $m+i\tan\theta$ is not always an integer, that is, there will be non-integer points. And these non-integer points do not exist in original image. This requires that we use the appropriate definition to give them values, and we call it as the process of interpolation. In the interpolation process, we limit the interpolation to the pixel which must be even set $x_e(m,n)$ in order to ensure perfect reconstruction. Interpolation through the following formula:

$$x(m+sign(i-1)\tan\theta,2n+i)=\sum_k \phi_k x(m+k,2n+i) \quad (2.2)$$

Where k and ϕ_k respectively represents point of $x_e(m+i\tan\theta,2n+i)$ in direction $i\tan\theta$ and interpolation coefficient. In this paper, we use Lagrange interpolation method to determine ϕ_k .

Similarly, during the update process operator U be changed into

$$U(d,m,n)=\sum_i \beta_i d(m+sign(i)\tan\theta,2n+i) \quad (2.3)$$

After above three steps, we can obtain a vertical high-pass sub-band H and a low-pass subband L, then constitute in 2-D direction lifting structure by doing horizontal 1-D direction lifting.

The selection of the angle θ is the key. First we choose the angle based on block, and the block size is 16×16. For each block, we select a pair of best directions of adaptive lifting (a vertical direction, a horizontal direction) to make the signal mostly smooth in this block. All pixels are lifting with the same direction in each block, and the direction of different block is to choose independently apart from the effects of other blocks. For an image, we need to select a pair of the best direction as lifting direction for each block. Finally the entire image energy as much as possible concentrated in the low-frequency sub-band(LL) after each block are lifted with their directions.

We first consider the choice of all the horizontal direction. We assume that the image I is cut into a series of 16 × 16 blocks, and symbol

$$B_i, i=1,2,3..., B_i\cup B_j=\varnothing(i\neq j), \cup B_i=I\ .$$

Further assume that the θ vertical direction of block B_i. We use the RD function to determine the best direction to lift for each block. First of all, we calculate the prediction error of the entire image.

$$D(I)=\sum_{B_i\in I}\sum_{m,n}|d_{B_i,\theta}(m,n)| \quad (2.4)$$

We can find that θ of B_i will independently impact value of $D(I)$. $D(I)$ can be taken as a function of a series $\theta_1,\theta_2,\theta_3......$ If making $D(I)$ the smallest value, block B_i must have the smallest error. Taking into account the coding of the direction of the various blocks need for additional information of bits, we adopt the following formula to determine the best direction to lifting.

$$RD(\vartheta)=D(I,\vartheta)+\lambda R(\vartheta) \qquad \vartheta=\theta_{B_1},\theta_{B_2},\theta_{B_3}...... \quad (2.5)$$

λ is a Lagrange multiplier, when the value in the horizontal lifting is 25 / 2, the value of vertical lifting is 5.

The method of choosing the vertical direction of each block is the same as horizontal, but all of our judgments are executed in the L sub-band after lifted and all sub-block sizes are changed into a 16 × 8 by sub-sampling. Its RD calculation algorithm is completely same as horizontal, so not too much described.

III. INTERPOLATION METHOD BASED ADAPTIVE DIRECTIONAL LIFTING WAVELET TRANSFORM

Image will be smoother if adjacent pixels have a more causality because lifting in the right direction. This characteristic allows us to predict each pixel in different lifting form to reflect more details other than the direction of the local characteristics.

The original adaptive (5,3) lifting wavelet only predicts with two points whose neighbor are the direction of pixels to be forecast. In fact, this is not good use of the strong causally between the pixels. Therefore, we use the Lagrange wavelet refers to [7][8]. Predicting pixels which be predicted with k points by using the Lagrange interpolation. So the mathematical expression of Lagrange operator P is:

$$P(x[n])=\sum_{i=1}^{k}\frac{\prod_{j\neq i}x[n_i](n-n_j)}{\prod_{j\neq i}(n_i-n_j)} \qquad (3.1)$$

We can find that we use some acquired pixels to predict pixels which are to be predicted from (3.1). So the larger the interpolation between pixels to predict and pixels to be predicted, the more close the value of predicting $P(x[n])$ to the original pixels $x[n]$.

So first we need to determine whether $x_o(m,n)$ is in the smooth area or in the shape area in order to select acceptable points to predict $x_o(m,n)$. We predict pixels by using dissymmetrical pixels in $x_o(m,n)$ both side.

$$P_1(x_o(m,n))=\sum_{i=0}^{k}\frac{\prod_{j\neq i}x(m+sign(i-1)\tan\theta,2n+i)(m-m_j)}{\prod_{j\neq i}(m_i-m_j)} \quad (3.2)$$

$$P_1(x_o(m,n))=\sum_{i=-k+1}^{1}\frac{\prod_{j\neq i}x(m+sign(i-1)\tan\theta,2n+i)(m-m_j)}{\prod_{j\neq i}(m_i-m_j)} \quad (3.3)$$

$P_1(x_o(m,n))$ and $P_2(x_o(m,n))$ separately represents non-symmetry points k +1 from $x_o(m,n)$ around to predict $x_o(m,n)$. Here, we take into account $x_o(m,n)$ possibility in the case of local minimum, so we add a value to the other side to predict more accurately. And as a result of a strong correlation of neighbor pixels, the k value can not be too large. So k value is 2 in this paper. From the two predict formula we can see that selected points may be the non-integer pixel. Therefore, we used the Lagrange interpolation method to interpolate these non-integer pixels.

Set a threshold δ to decide $x_o(m,n)$ whether it is in a smooth or non-smooth area after obtain $P_1(x_o(m,n))$ and $P_2(x_o(m,n))$. That is $x_o(m,n)$ in the smooth area if $|P_1(x_o(m,n))-P_2(x_o(m,n))|<\delta$, then using the following formula to predict.

$$P_1(x_o(m,n))=\sum_{i=-k+1,i\neq 0}^{k}\frac{\prod_{j\neq i}x(m+sign(i-1)\tan\theta,2n+i)(m-m_j)}{\prod_{j\neq i}(m_i-m_j)} \quad (3.4)$$

On the contrary, $x_o(m,n)$ is in the non-smooth area and reconsider its forecast for the selection of side of pixels associated with higher value. Taking off the pixel of other side, only consider one side to predict $x_o(m,n)$ again, and will obtain two new predicted values $P_3(x_o(m,n))$ and $P_4(x_o(m,n))$. For the perfect reconstruction, we can not directly compare two predicted values and $x_o(m,n)$, while comparing two values which are the most close $x_o(m,n)$ ($x_e(m,n+\tan\theta)$ and $x_e(m+1,n-\tan\theta)$):

$$\delta_1=|x_e(m,n+\tan\theta)-P_3(x_o(m,n))| \qquad (3.5)$$

$$\delta_2=|x_e(m+1,n-\tan\theta)-P_4(x_o(m,n))| \qquad (3.6)$$

Compare δ_1 and δ_2, and obtain predict operator using by more correlation side.

$$P(x_o(m,n)) = \begin{cases} P_3(x_o(m,n)) & \delta_1 < \delta_2 \\ P_4(x_o(m,n)) & \delta_1 > \delta_2 \end{cases} \quad (3.7)$$

IV. Experiment Result

Transform image by wavelet using interpolation method based on adaptive directional lifting construction is proposed in this paper. Then SPIHT encode technology is used, and compared result with the (5,3) wavelet finally. We use two images for tests: Barbara and Lena.

We first decompose Barbara with the more obvious characteristics of the local geometry of space by lifting wavelet.

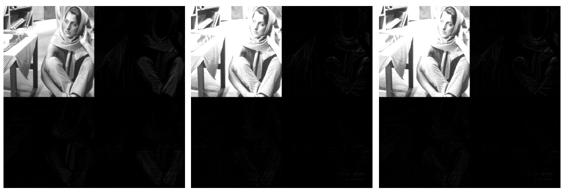

Figure 3. (5,3)lifting wavelet, adaptive directional wavelet, interpolation for Barbara first level decomposition

According to the comparison of low-frequency and high frequency image of three methods in these images, we find that coefficient of direction adaptive lifting structure is not only much less than the (5,3) wavelet in high-frequency sub-band, but also the interpolation method retains most of the details of information, and its image is more clear and more visible in the pants and striped tablecloths compared with non-use of interpolation method in the low-frequency images. Following are accounting for the various sub-band taking part of entire image of the various methods.

TABLE I. Barbara sub-band take part of entire image

	LL	LH	HL	HH
5-3 wavelet	98.11%	1.46%	0.19%	0.22%
Adaptive directional	99.73%	0.18%	0.06%	0.01%
Interpolation	99.76%	0.16%	0.04%	0.01%

From the above table, we can see that image energy becomes more concentrated after interpolation method lifting. So that succeed encode can be more effective. The below table is result which is compared the PSNR after image compression. We transform image with three levels and SPIHT encode.

From the table, we can find that PSNR of the adaptive directional lifting construction is higher than the (5,3) lifting wavelet. And the interpolation construction method is better than that not using interpolation method in most cases.

TABLE II. Barbara PSNR of after compression

Compression ratio	0.02	0.05	0.08	0.1	0.5
5-3 wavelet	19.15	23.35	25.93	29.57	48.71
Adaptive directional	19.19	24.19	27.20	30.48	52.54
Interpolation	19.20	24.30	27.24	30.63	52.28

TABLE III. Lena PSNR of after compression

Compression ratio	0.02	0.05	0.08	0.1	0.5
5-3 wavelet	20.14	27.69	33.03	35.92	52.83
Adaptive directional	20.20	27.88	33.27	35.85	52.72
Interpolation	20.20	27.88	33.40	35.99	52.91

V. Conclusion

This paper proposes an interpolation method based on adaptive directional lifting wavelet construction. This construction can adapt image local characteristic to the lifting transform. From experimental result, it improves the tradition lifting wavelet construction and the PSNR. So it strengthens image compression performance. But it will take large time during selecting adaptive direction due to considering RD of every angle. This problem will be studied and solved in the future work.

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