DRIMUX: Dynamic Rumor Influence Minimization with User Experience in Social Networks

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Abstract—With the soaring development of large scale online social networks, online information sharing is becoming ubiquitous everyday. Various information is propagating through online social networks including both the positive and negative. In this paper, we focus on the negative information problems such as the online rumors. Rumor blocking is a serious problem in large-scale social networks. Malicious rumors could cause chaos in society and hence need to be blocked as soon as possible after being detected. In this paper, we propose a model of dynamic rumor influence minimization with user experience (DRIMUX). Our goal is to minimize the influence of the rumor (i.e., the number of users that have accepted and sent the rumor) by blocking a certain subset of nodes. A dynamic Ising propagation model considering both the global popularity and individual attraction of the rumor is presented based on realistic scenario. In addition, different from existing problems of influence minimization, we take into account the constraint of user experience utility. Specifically, each node is assigned a tolerance time threshold. If the blocking time of each user exceeds that threshold, the utility of the network will decrease. Under this constraint, we then formulate the problem as a network inference problem with survival theory, and propose solutions based on maximum likelihood principle. Experiments are implemented based on large-scale real world networks and validate the effectiveness of our method.

Index Terms—Social network, rumor blocking, survival theory.

1 INTRODUCTION

With the soaring development and rising popularity of large-scale social networks such as Twitter, Facebook, and Chinese Sina Weibo, etc., hundreds of millions of people are able to become friends [2] and share all kinds of information with each other. Online social network analysis has also attracted growing interest among researchers [3], [4], [5], [6]. On one hand, these online social platforms provide great convenience to the diffusion of positive information such as new ideas, innovations, and hot topics [7], [8]. On the other hand, however, they may become a channel for the spreading of malicious rumors or misinformation [9], [10], [11]. For example, some people may post on social networks a rumor about an upcoming earthquake, which will cause chaos among the crowd and hence may hinder the normal public order. In this case, it is necessary to detect the rumor source and delete related messages, which may be enough to prevent the rumor from further spreading. However, in certain extreme circumstances such as terrorist online attack, it might be necessary to disable or block related Social Network (SN) accounts to avoid serious negative influences. For instance, in 2016, the families of three out of the forty nine victims from the Orlando nightclub shooting incident filed a lawsuit against Twitter, Facebook and Google for providing “material support” to the terrorism organization of the Islamic State of Iraq and Syria (ISIS) [12]. These companies then took measures to block related accounts, delete relevant posts and fanpages on their social network platforms to prevent the ISIS from spreading malicious information. Additionally, Facebook et al. also have issued relevant security policies and standards to claim the authority to block accounts of users when they are against rules or at risk [13]. Undoubtedly, malicious rumors should be stopped as soon as possible once detected so that their negative influence can be minimized.

Most of the previous works studied the problem of maximizing the influence of positive information through social networks [14], [15], [16]. Fast approximation methods were also proposed to influence maximization problem [17], [18]. In contrast, the negative influence minimization problem has gained much less attention, but still there have been consistent efforts on designing effective strategies for blocking malicious rumors and minimizing the negative influence. Budak et al. [9] introduced the notion of a “good” campaign in a social network to counteract the negative influence of a “bad” one by convincing users to adopt the “good” one. Kimura et al. [19] studied the problem of minimizing the propagation of malicious rumors by blocking a limited number of links in a social network. They provided two different definitions of contamination degree and proposed corresponding optimization algorithms. Fan et al. [20] investigated the least cost rumor blocking problem in social networks. They introduced the concept of “protectors” and try to select a minimal number of them to limit the bad influence of rumors by triggering a protection cascade against the rumor cascade. However, there are a few limitations in those works. First, they consider the rumor popularity as constant during the whole propagation
process, which is not close to the realistic scenarios. Second, in the design of the rumor blocking strategies, either blocking nodes or links, they fail to take into account the issue of user experience in real world social networks. We have to avoid blocking the accounts of users for such a long time that they may lodge complaints or even quit the social network. Therefore, it is necessary to consider the impact of blocking time on both individual node and the entire network.

In this paper, we investigate the problem of dynamic rumor influence minimization with user experience. First, based on existing works on information diffusion in social networks [21], [22], [23], [24], we incorporate the rumor popularity dynamics in the diffusion model. We analyze existing investigations on topic propagation dynamics [25] and bursty topic patterns [26]. Then we choose Chi-squared distribution to approximate the global rumor popularity. Inspired by the novel energy model proposed by Han et al. [27], we then analyze the individual tendency towards the rumor and present the probability of successful rumor propagation between a pair of nodes. Finally, inspired by the concept of Ising model [28], we derive the cooperative succeeding probability of rumor propagation that integrates the global rumor popularity with individual tendency. After that, we introduce the concept of user experience utility function and analyze the impact of blocking time of nodes to the rumor propagation process. We then adopt the survival theory to explain the likelihood of nodes getting activated, and propose both greedy and dynamic algorithms based on maximum likelihood principle.

The contributions of our work are as follows:

• We propose a rumor propagation model taking into account the following three elements: First, the global popularity of the rumor over the entire social network, i.e., the general topic dynamics. Second, the attraction dynamics of the rumor to a potential spreader, i.e., the individual tendency to forward the rumor to its neighbors. Third, the acceptance probability of the rumor recipients. In our model, inspired by the Ising model, we combine all three factors together to propose a cooperative rumor propagation probability.

• In our rumor blocking strategies, we consider the influence of blocking time to user experience in real world social networks. Thus we propose a blocking time constraint into the traditional rumor influence minimization objective function. In that case, our method optimizes the rumor blocking strategy without sacrificing the online user experience.

• We use survival theory to analyze the likelihood of nodes becoming activated or infected by the rumor before a time threshold which is determined by the user experience constraint. Then we propose both greedy and dynamic blocking algorithms using the maximum likelihood principle.

The rest of the paper is organized as follows. In Section 2, we introduce the preliminaries of social network and information diffusion models. Next we give an overview of the related work in Section 3. Then we propose the problem formulation in Section 4, the solutions in Section 5, and the experiments in Section 6. Finally, we conclude the paper in Section 7.

2 PRELIMINARIES

2.1 Social Network Definition

A social network, in mathematical context, can be formulated as a directed graph \( G = (V, E) \) consisting of a set of nodes \( V \) representing the users, and a set of directed edges \( E \) denoting the relationship between users (e.g., following or being followed). Figure 1 shows the random graph illustration of a social network. Let \( |V| = N \) denote the number of nodes, and \( (u, v) \in E \) denote the directed edge from node \( u \) to node \( v \) \((u, v \in V)\), and \( \alpha_{uv} \in \{0, 1\} \) denote the edge coefficient, where \( \alpha_{uv} = 1 \) represents the existence of edge \((u, v)\), and \( \alpha_{uv} = 0 \), otherwise. We use \( p_{uv} \) to denote the probability of \( u \) sending the rumor to \( v \) and \( v \) accepting it, i.e., the success probability of \( u \) activating \( v \). Let \( D(u) \) denote the in-degree of node \( u \). From Figure 1, we can see nodes in larger size have higher degree than those in smaller size. The degree of a node is also an indication of “influence” in a social network since higher degree denotes more connections to other nodes, thus it implies more opportunities to share information (both positive and negative) with other nodes.

2.2 Rumor Diffusion Model

Rumor diffusion mechanism is similar with that of epidemic propagation [29]. During the propagation of rumors, each node could have one of the following three states: Susceptible (S), Infected (I) and Recovered (R), which is known as the SIR model [30], [31]. The state of being susceptible represents the node has the potential to accept and spread the rumor at any time; Infected indicates the node has already accepted and spread the rumor; Recovered denotes the state of the node identifying the rumor and denying it. In this paper, we consider the rumor propagation as a progressive process, i.e., once a node is infected, it will stay infected and not recover, which is the SI model.
Diffusion models describe the process of information propagating through the network. Two classic diffusion models are the Linear Threshold (LT) [32] and the Independent Cascade (IC) model [33], [34]. In LT model, an inactive node $u$ becomes activated if the ratio of its activated parent nodes surpasses a certain pre-defined threshold $0 < \theta < 1$. In this paper, since we mainly focus on pairwise probabilistic rumor propagation among nodes including individual tendency of a node to a rumor as well as the global popularity probability of a rumor, it is more suitable to adopt the IC model in our work.

The IC model has been widely adopted in information diffusion problems. The whole propagation process proceeds in discrete time steps $t_0, t_1, t_2, \ldots$. Initially, the cascade is triggered by a set of activated nodes, i.e., the seed nodes at $t_0$. In our rumor diffusion context, we assume the rumor is started by one source node $s$ in the network, and the other nodes are inactive. We use $s_u(t) \in \{0, 1\}$ to denote the state of node $u$ at time step $t$, where $s_u(t) = 1$ represents $u$ is activated and $s_u(t) = 0$, otherwise. For every following time step $t \geq 1$, each node $u$ which was activated at time step $(t - 1)$ has a single opportunity to activate any of its currently inactive neighbors $v$ with a success probability $p_{uv}$. In our context, it means in each time step, any node that has accepted the rumor in previous time step has a chance to let their inactive neighbors accept the rumor. For simplicity, we assume that once a node is activated by the rumor, it will stay activated until the end of the diffusion process.

### 3 RELATED WORK

#### 3.1 Topic Dynamics

Researchers have studied the temporal dynamics of web topics based on real-world statistics. Yang et al. [25] analyzed how the number of tweets related to a specific theme (i.e., the popularity of a topic) changes with time, and revealed that a topic evolution generally consists of three phases, i.e., a rising phase from the start, a peak period and then a fading phase. Fluctuations in each phase may result in different temporal characteristics. Yang et al. [25] proposed $K$-Spectral Centroids clustering algorithm for classifying online content according to their temporal patterns and finally extract six representative patterns from million-scale tweets and blog posts. Crane et al. [35] demonstrated the existence of Poisson distribution and Power-law relaxation in controlling the topic evolution over time. Figure 2 shows several typical topic evolution curves based on the data extracted from Sina Weibo. From the figure, in each topic evolution curve, we can see the three phases mentioned above. It is also discernable that every topic has a certain lifespan of its own, which is similar to the case of rumor propagation process. Thus, it is reasonable to utilize a function to simulate the process of rumor propagation. According to the topic evolution characteristics, in our work, we use the Chi-squared distribution [36], [37], [38] to simulate the rumor propagation dynamics.

#### 3.2 Energy Model

Rumor propagation can be considered as a type of social contagion process [39] with several special characteristics. Firstly, people’s interest of a rumor tends to decrease with time, which indicates the probability of a node willing to forward the rumor. That process is similar to the simulated annealing process [40]. Han et al. [27] proposed a novel energy model to describe the rumor propagation process. They introduce the heat energy calculation formula $\Delta E = cmT \Delta T$ in Physics to analogize the rumor impact. The rumor’s influence on individual node is formulated as the amount of accumulated heat energy. Based on the model proposed by Han et al. [27], we define the expression of individual tendency with respect to the success activation probability between a pair of nodes. In addition, even though an activated node does transmit the rumor to its neighbors, the probability of these neighbors accepting the rumor is still to be determined. In that case, we can define the acceptance probability of the rumor recipient. By combining the rumor sending probability at the transmitting end with the rumor acceptance probability at the receiving end, we can obtain the ultimate rumor propagation probability.

#### 3.3 Ising Model

The Ising model [41] is a widely applicable model in the research of Physics theory. It is a simple theoretical de-
scription of the concept of ferromagnetism in Physics [42]. Specifically, it describes the phenomenon that when an array of atomic spins align in the way that the magnetic moments associated to them will all point in the same direction. Then it will create a macroscopic magnetic moment. Generally speaking, the Ising model contains two parts – the microscopic and macroscopic parts. The microscopic part represents the local or individual behavior which is the alignment of each of the atomic spins. Correspondingly, the macroscopic part stands for the global or collective behavior which is the exterior magnetic moment. Based on its intrinsic attributes, the Ising model can be generalized to other similar scenarios. In our work, we utilize it to model the rumor propagation process in social networks.

3.4 User Experience

User experience is an important factor for various services including social networks [43]. Existing rumor blocking strategies block either nodes (users) or links (connections between users) in social networks to prevent the rumor from further propagation. However, none has analyzed the impact of blocking nodes. Generally speaking, the longer the user is blocked, the less satisfactory the user feels about the social network. Therefore, if the blocked time surpasses a certain threshold, it is possible that the user may quit the social network or at least lodge a complaint to the administrator. Bhatti et al. [44] analyzed the user-perceived quality in web server design and found that users’ tolerance for latency decreases over the duration of interaction with a site. A utility function was presented to measure the customer satisfaction. Inspired by that, in our work, we apply a modified utility function to measure user experience in rumor blocking.

3.5 Rumor Influence Minimization

Rumor influence minimization addresses the problem of minimizing the propagation effect of undesirable rumors in social networks. Figure 3 demonstrates the mechanism of rumor blocking in social networks. It shows both the normal rumor propagation process without any blockage in the social network and the process blocking a set of nodes on the path of rumor propagation. The rumor influence minimization problem is converse to the classic influence maximization problem [14]. It has been investigated in different influence diffusion models in social networks. Fan et al. [20] studied the least cost rumor blocking problem in social networks, and introduced the notion of “protectors” to limit the bad influence of rumors by initiating a protector cascade to propagate against the rumor cascade. Greedy algorithm is proposed for both opportunistic and deterministic cascade models. However, Kimura et al. [19] proposed the strategy of blocking links instead of nodes in social networks so as to minimize the propagation of malicious rumors. Different contamination minimization problems are defined based on different definitions of contamination degree of a network.

4 PROBLEM FORMULATION

4.1 Dynamic Rumor Propagation with Ising Model

Kempe et al. [14] considered the success probability $p_{uv}$ as a system parameter and is fixed at the very beginning of the cascade. However, based on the topic dynamics we discussed in a previous section, at different time steps of the propagation process, a topic can vary dramatically in its popularity. Besides, the rumor attraction [27] for each individual node $u \in V$ is also a realistic factor we should take into account. That means the success of rumor propagation between neighbors includes two aspects: first, the activated node $u$ has to be so attracted by the rumor that it will choose to send the rumor to its neighbors; second, one of $u$’s inactive neighbors $v$ decides to accept the rumor. Only after those two steps, we can claim that $v$ is activated. In other words, the success of rumor propagation depends both on the global popularity and the individual tendency of the rumor topic, which can be regarded as a generalized feature of the Ising model.

Now we investigate the two steps of a successful rumor propagation. In the first step, at any time stamp $t_j$, $u$ is one of the activated nodes in time stamp $t_{j-1}$. Based on the work in [27], we give the modified version of the probability of node $u$ sending the rumor to one of its inactive neighbors $v$ as

$$p_{uv}^\text{send}(t_j) = \frac{p_0}{\lg(10 + t_j)},$$

where $p_0$ is the initial sending probability at time stamp 0. On the receiving end, the probability of node $v$ accepting the rumor transmitted by its parent node $u$ is also given as

$$p_{uv}^\text{acc} = 1/D_v,$

where $D_v$ is the in-degree of node $v$. That denotation can be interpreted as that if a node has very high degree, it will receive more information than those with lower degrees. In that case, due to the large number of pieces of information it receives, every single piece of information will have much lower possibility to be read, recognized and forwarded by the node. For example, if a user has only one friend on a social network, he or she would probably read all the messages forwarded by his or her friend, and thus if the user finds a rumor interesting, he or she will probably believe and forward it. In contrast, if a user has hundreds or even thousands of friends, he or she could easily miss most of the information.

Thus, based on the above analysis, we then give the probability of successful rumor propagation from $u$ to $v$ as

$$p_{uv}^\text{end} \cdot p_{uv}^\text{acc} = \frac{p_0}{D_v \cdot \lg(10 + t_j)},$$

which can be defined as the individual tendency between different pair of nodes in the network.

Now we discuss the global topic popularity of the rumor. As mentioned in related work, the rumor popularity generally includes three phases and approximately subject to the chi square distribution, which is given by

$$p_{\chi^2}(t; k) = \frac{2^{(\frac{k}{2})} t^{k-1} e^{-\frac{t^2}{2}}}{\Gamma(\frac{k}{2})},$$
where \( k > 0 \) represents the degree of freedom, \( \Gamma(\cdot) \) is the Gamma function. It explains a common social phenomenon that when a rumor spreads for a while, it may create a “rumor atmosphere” that could affect the judgements or decisions of users on online social networks.

According to the Ising model [28], the “phase transition” of a spin involves both short-range interaction with its nearest neighbors and long-term system evolution, and is a combined result. Inspired by that, we propose the cooperative propagation probability integrating \( p_{gb}(t; k) \) with \( p_{ind}(t) \) in the form of a logistic function as

\[
p_{uv}(t) = \frac{1}{1 + \exp \left[ -\beta_1 \cdot p_{gb}(t; k) \cdot \beta_2 \cdot p_{ind}(t) \right]}
\]

where \( \beta_1, \beta_2 \in (0, 1) \) are the balance coefficients which satisfy \( \beta_1 + \beta_2 = 1 \).

Based on this cooperative propagation probability, the probability of node \( v \) getting activated at time stamp \( t_j \) can be given by

\[
P_r[s_v(t_j) = 1] = 1 - \prod_{u \in P_v} [1 - s_u(t_j-1)p_{uv}(t_j)],
\]

where \( P_r[\cdot] \) represents probability, and \( P_v \) represents the parent nodes of \( v \).

### 4.2 User Experience Utility

In the formulated optimization problem, one important constraint condition is the user experience utility function. Therefore, before giving the concrete algorithm, first, we elaborate on the user experience utility function.

It is a common sense that user experience is a critical element in the success of modern business. A large variety of communication services involve user experience, such as web searching, telephone connecting, etc. For customers using those services, the latency plays a tremendous role in their satisfaction extent, i.e., the user experience utility. Specifically, in our proposed problem, the user experience utility lies in the blocked time \( t^b_u \) of the selected node \( u \).

In our work, we try to find the proper user experience utility function to characterize the impact of nodes being blocked to the entire social network. Specifically, we analyze both the homogeneous and heterogeneous scenarios.

#### Homogeneous Networks

For homogeneous social networks, the simple case, we assume that all the nodes have the same blocked time threshold \( T_{th} \). In other words, we assume that all the users in the same social network have the same tolerance of the time being blocked regardless of the time they have been in the social network. In this scenario, based on the user experience definition on web server in [44], we define the user experience utility function as

\[
U_{het} = \frac{1}{N} \sum_{u=1}^{N} \frac{T_{th} - T_{b}(u)}{T_{th}},
\]

where \( T_{th} \) is used to record the blocked time of node \( u \) in the whole propagation process.

#### Heterogeneous Networks

In heterogeneous social networks, different nodes have distinct properties. For example, in real world social networks like Twitter or Weibo, different users have different levels according to the time they have spent on it, the number of followers they have or the number of messages they have posted. Typically, we can simply divide all users into VIPs and ordinary users considering their levels. Intuitively, the social network operators would try to avoid blocking the VIP users as possible as they can because of the impact they have on the enormous followers and hence on the entire network. On the other hand, from the perspective of the VIPs, since they usually have frequent interactions with their followers, there is a high possibility that they can not tolerate to be blocked for a long time. As a result, the VIPs usually have a relatively low tolerance threshold of the blocked time.

Let \( L(u) \) denote the level of node \( u \), and \( T(u) \) denote the tolerance threshold of node \( u \) when it is blocked. Then we propose the following expression:

\[
T(u) = \frac{1}{L(u)}.
\]

The mechanism can be explained as follows. When the level of a user \( u \), \( L(u) \), is approximate to zero, i.e., the user has just joined the social network, its tolerance threshold is close to infinity, because even if it is blocked forever, it can just apply for a new account without any loss. On the contrary, if \( u \) is a VIP user, the higher its level is, the less blocked time it can endure. When \( L(u) \) is large enough, its tolerance will be asymptotic to zero.

There are several metrics to define the level or significance of a node in a social network, such as degree, eigenvector or betweenness. Also there is a widely adopted PageRank algorithm [45] which can be regarded as a variant of eigenvector centrality. For simplicity, we here use the degree of a node \( u \), \( D(u) \), to represent its level, i.e., \( L(u) = \zeta D(u) \), where \( \zeta \) is a constant coefficient. Thus, the user experience utility in a heterogeneous network can be written as

\[
U_{het} = \frac{1}{N} \sum_{u=1}^{N} (1 - \zeta D(u) T_b(u)).
\]
4.3 Objective Formulation

Now our goal is to minimize the influence of a rumor as much as possible (e.g., minimize the number of activated nodes at the end of propagation process) under the constraint of user experience utility. We formulate the DRIMUX problem as follows:

$$\min \ E[ \sum_{v \in V} s_v(T) ]$$

s. t. $U_{\text{hom}}(U_{\text{net}}) \geq U_{\text{th}},$

where $s_v(T)$ represents the state of a node $v$ by end of a time interval $T$ as denoted in Equation (6). $E[ \sum_{v \in V} s_v(T) ]$ denotes the expected number of activated nodes by the end of $T$. The constraint condition restrains the user experience above a threshold $U_{\text{th}}$. In the following section we will discuss how the user experience constraint affects the activation likelihood of each node.

5 Proposed Solutions

In this section, we analyze the DRIMUX optimization problem from the perspective of a network inference problem with survival theory and then propose the greedy algorithm and dynamic blocking algorithm based on different nodes selection schemes and the maximum likelihood principle.

5.1 Survival Theory

In this section, we analyze the likelihood of nodes getting activated during each time slot in the process of rumor propagation using the Survival Theory. Firstly, in our model, we assume that a rumor has been spreading for some time before it is detected at time $t_0$. Specifically, we assume that when the ratio of the infected nodes reaches a certain threshold, it would draw the attention of the monitoring department and then be detected whether it is a rumor or not. If it is a rumor, relevant blocking strategies would be initiated to block it. Mathematically, we use $I(t)$ to denote the ratio of infected nodes in the social network at time $t$.

It is also assumed that by time $t_0$, there have already been a total number of $N_1$ activated nodes, and $N_2 = N - N_1$ nodes remain inactive. Let $V_{N_1}$ and $V_{N_2}$ denote the set of activated and inactive nodes at time $t_0$ respectively. Therefore, from $t_0$ on, the system can be viewed as $N_1$ independent cascades propagating through the network, and our goal is to select $K$ nodes from $N_2$ and block them so that the final number of activated nodes during the observation time window $T$ can be minimized.

Let $C = \{c_1, \ldots, c_{N_1} \}$ denote the set of cascades triggered by $N_1$ activated nodes by time $t_0$. A cascade $c_i \in C$ can be represented by a $N$-dimensional time vector $t^{c_i} = (t_1^{c_i}, \ldots, t_N^{c_i})$, where $t_j^{c_i} \in [t_0, t_0 + T] \cup \{\infty\}$, $j = 1, 2, \ldots, N_2$ is the activated time of node $j$ in cascade $c_i$. The observation time window $T$ is decided by the user experience utility constraint mentioned in (10), and $\infty$ means the node is not activated until the end of the observation time ($t_0 + T$).

Here, we first consider the propagation process of only one cascade and then the results can be extended to the scenario of multiple cascades.

5.1.1 Survival Function

In order to analyze the likelihood of nodes getting activated during one time slot in a cascade, we adopt the Survival Theory to calculate the probability of a single node $v$ getting activated in a given time period. First, we introduce the survival function defined as [46]

$$S(t) = \Pr(t < T),$$

where $T$ is a continuous random variable representing the occurrence time of an event of interest, $t$ is a specified constant. The survival function represents the probability that the event of interest occurs after the observation “deadline”. If we use the terminology “death” to represent the occurrence of the event, we can claim that the target “survives” if its occurrence takes place after the specified time $t$. Then we have the cumulative distribution function $F(t)$:

$$F(t) = \Pr(T \leq t) = 1 - S(t).$$

Accordingly, the probability density function $f(t)$ is given by

$$f(t) = \frac{d}{dt} F(t) = -S'(t).$$

Alternatively, there is another method named hazard rate to express the instant activation rate of node $v$. Specifically, the hazard rate indicates the probability of a single node with current state $s(t)$ getting activated at time $(t+dt)$. We define $\alpha_v(t|s(t))$ as the hazard rate of node $v$ conditioned on the set of nodes activated by time $t$. Our goal now is to analyze the impact of the hazard rate of different nodes to the rumor influence minimization problem.

5.1.2 Hazard Rate

Based on the above analysis, the hazard rate can be viewed as an alternative interpretation of the distribution of $T$, which characterizes the instantaneous rate of occurrence of an event. It is defined as:

$$\alpha_v(t|s(t)) = \lim_{dt \to 0} \frac{\Pr(t \leq T \leq t + dt | T > t)}{dt}$$

$$= \lim_{dt \to 0} \frac{\Pr(t \leq T \leq t + dt)}{\Pr(T > t)dt}$$

$$= \lim_{dt \to 0} \frac{F(t + dt) - F(t)}{S(t)dt}$$

$$= \lim_{dt \to 0} \frac{f(t)dt}{S(t)} = \frac{f(t)}{S(t)},$$

where $S'(t)$ is the derivative of $S(t)$. $\Pr(t \leq T \leq t + dt | T > t)$ denotes the conditional probability that the event of interest will occur in time period $[t, t+dt]$ given that it has not occurred before time $t$.

Accordingly, we can have

$$S(t) = e^{-\int_0^t \alpha_v(\tau|s(\tau))d\tau},$$

and for a certain node $v$, according to (12), we have

$$F_v(t|s(t)) = 1 - e^{-\int_0^t \alpha_v(\tau|s(\tau))d\tau}.$$
5.2 Problem Solutions

Based on the survival analysis, we propose an additive survival model where we assume that the probability of node \( v \) getting activated is the weighted summation of the propagation probabilities mentioned in (5) of all the previously activated nodes set \( \{ u : t_u < t \} \). Thus, in our context, the hazard rate is given by

\[
\alpha_v(t|s(t)) = \alpha_v^T s(t) = \sum_{u \in N, u < t} \alpha_{uv} p_{uv}(t), \tag{17}
\]

where \( \alpha_v = (\alpha_{uv}), u = 1, 2, \ldots, N \) is a non-negative parameter vector indicating the existence of the edge between node \( u \) and \( v \). \( \alpha_{uv} = 1 \) if there is an edge between them; and \( \alpha_{uv} = 0 \), otherwise.

We then define a coefficient matrix \( A := [\alpha_v] \in \mathbb{R}_+^{N \times N} \) to denote the structure of the network in terms of the connection between any pair of nodes in the network. Let \( A_0 \) be the original network coefficient matrix before any nodes are blocked. By substituting \( \alpha_v(\tau|s(\tau)) \) in (16) with (17), then we can have:

\[
F_v(t|s(t)) = 1 - e^{-\sum_{u < t} \alpha_{uv} p_{uv}(\tau) d\tau} = 1 - e^{-\sum_{u < t} \alpha_{uv} f_{uv}^1 p_{uv}(\tau) d\tau} = 1 - e^{-\sum_{u < t} \alpha_{uv} f_{uv}^1 p_{uv}(\tau) d\tau} = 1 - e^{-\sum_{u < t} \alpha_{uv} f_{uv}^1 p_{uv}(\tau) d\tau} \tag{18}
\]

Accordingly, we have the likelihood function of the activation of node \( v \), \( f_v(t|s(t)) \), as following:

\[
f_v(t|s(t)) = \frac{dF_v(t|s(t))}{dt} = \sum_{u \in N, u < t} \alpha_{uv} p_{uv}(t) \prod_{\varphi < t} e^{-\alpha_{uv} f_{\varphi}^1 p_{uv}(\tau) d\tau}. \tag{19}
\]

Given the activation likelihood of a single inactive node \( v \in V_{\bar{N}} \), now we consider any number of inactive nodes in a cascade, during the entire observation window \( T \), \( t \leq T = (t_1, \ldots, t_i, \ldots, t_N | 0 \leq t_i \leq t_0 + T) \). We assume that every activation is conditionally independent on activations occurring later given previous activations. Then we can compute the activation likelihood as:

\[
f(t^{<T}; A) = \prod_{t_i < T} \sum_{u \in N, u < t_i} \alpha_{uv} p_{uv}(t_i) \times \prod_{\varphi < t_i} e^{-\alpha_{uv} f_{\varphi}^1 p_{uv}(\tau) d\tau}. \tag{20}
\]

Based on the activation likelihood function, we then design the blocking algorithms. First, we choose to select and block all \( K \) nodes at the same time \( t_0 \). As is shown in Eq. (20), the activation likelihood of an inactive node \( v \) is related to the hazard rate coming from all previously activated nodes. Therefore, the early activated nodes play a significant role in the entire process. Hence, we propose the following greedy algorithm to minimize the influence of the rumor within one time stamp after it is detected. We assume that there are \( M \) time steps: \( t_1, \ldots, t_M \) during the whole observation window \( T \), with each time step lasting \( T/M \).

5.2.1 Greedy Algorithm

The proposed Greedy algorithm tries to block the rumor as fast as possible to prevent the rumor from further propagation. The working mechanism is as following: At time \( t_0 \) when we detect the rumor, we immediately select all \( K \) nodes in our budget and block them (i.e., remove all the links of it so that it can not communicate with its neighbors). Mathematically, the Greedy algorithm aims to minimize the likelihood of inactive nodes getting activated at \( t_i \), i.e., the next time stamp after the rumor is detected. The likelihood of nodes getting activated at time \( t_i \) is given by

\[
f(t_1|s(t_0)) = \prod_{v \in V_{\bar{N}}: u < t_0} \sum_{u \in N, u < t_0} \alpha_{uv} p_{uv}(t_1) \times \prod_{\varphi < t_0} e^{-\alpha_{uv} f_{\varphi}^1 p_{uv}(\tau) d\tau}. \tag{21}
\]

Correspondingly, the objective function is

\[
\min_{A} \{ f(t_1|s(t_0)) \}
\]

s. t. \( \alpha_{uv} \in \{0, 1\} \).

Then, the greedy algorithm is presented as below:

Algorithm 1 Greedy Algorithm

Input: Initial Edge matrix \( A_0 \).

Initialization: \( V_B = \emptyset \).

for \( i = 1 \) to \( K \) do

\[ u = \arg \max_{v \in V_{\bar{N}}} \{ f(t_1|s(t_0); A_{i-1}) - f(t_i|s(t_0); A_{i-1}\backslash \{u\}) \} \]

\[ A_i := A_{i-1}\backslash \{u\} \]

\[ V_B = V_B \cup \{u\} \]

end for

Output: \( V_B \).

5.2.2 Dynamic Blocking Algorithm

Different from the greedy blocking algorithm, which is a type of static blocking algorithm, we propose a dynamic rumor blocking algorithm aiming to incrementally block the selected nodes instead of blocking them at once. In that case, the blocking strategy is split into several rounds and each round can be regarded as a greedy algorithm. Thus, how to choose the number of rounds is also very important for the algorithm. In the following part, we will elaborate on the algorithm design and how we choose the specific parameters. From the probabilistic perspective, we seek to formulate the likelihood of inactive nodes becoming activated in every round of rumor blocking. Correspondingly, the likelihood function is given by

\[
f(t_j|s(t_{j-1})) = \prod_{v \in V(t_j): u < t_j} \sum_{u \in N, u < t_j} \alpha_{uv} p_{uv}(t_j) \times \prod_{\varphi < t_j} e^{-\alpha_{uv} f_{\varphi}^1 p_{uv}(\tau) d\tau}, \tag{23}
\]

where \( V(t_j) \) represents the set of nodes which remain inactive at time stamp \( t_j \).

Different from the case of greedy algorithm, the objective function of the dynamic blocking algorithm is implemented in several rounds. In each round, it is similar to equation (22). We assume that there are \( n \) rounds of blocking in
It is noticeable that the proposed greedy algorithm (22) is a special case of the dynamic blocking algorithm (24) with \( n = 1 \). Accordingly, the dynamic blocking algorithm can be presented as following:

**Algorithm 2 Dynamic Blocking Algorithm**

- **Input:** Initial Edge matrix \( A_0 \).
- **Initialization:** \( V_B(t) = \emptyset \).
- **for** \( j = 1 \) to \( n \) **do**
  - **for** \( i = 1 \) to \( k_j \) **do**
    - \( \Delta_j = f(t_j | s(t_j - 1); A_{i-1}) - f(t_j | s(t_j - 1); A_{i-1} \setminus u) \)
    - \( u = \arg \max_{v \in V} (\Delta_j) \)
    - \( A_i := A_{i-1} \setminus u \)
    - \( V_B(t_j) = V_B(t_j) \cup \{ u \} \)
  - **end for**
- **end for**
- **Output:** \( V_B(t) \).

The dynamic blocking algorithm runs as follows: at the very first stage of blocking, we select a number \( k_1 \) nodes to block based on the Edge matrix and previously infected nodes; in the next round, we move forward with the rumor diffusion, and then use the updated status to block additional \( k_2 \) nodes. The blocking process continues at each following instants until the budget runs out at a moment \( t_n \), which can be expressed as \( \sum_{j=1}^{n} k_j = K \). In real implementation, we decrease \( k_j \) as time goes by, and a practical example is \( k_j = 2^{(-j)} * K \). Instead of blocking \( K \) candidates at the moment of detection, as previous static blocking strategies do, this dynamic approach is carried out in a progressive way. The design philosophy is to take advantage of instantaneous information all along the dissemination, since this the activation likelihood of a given moment is a variable which depends on the temporal Edge matrix and previous status. Rather than sparing all the efforts at once, we apply consequent force to block the diffusion of rumors. In this way, the global efficiency outweighs the previous static decisions.

**6 TIME COMPLEXITY ANALYSIS**

In this section, based on the above algorithms we propose, we present the time complexity analysis of different algorithms.

**Proposition 1.** The time complexity of the classic greedy algorithm, the proposed greedy algorithm and the proposed dynamic blocking algorithm is \( O(K|E|) \), \( O(K|E||V|) \) and \( O(K|E||V|) \) respectively, where \( K << |E| \).

**Proof 1.** According to the algorithms we give in the original version of our paper, the analysis of their time complexity is given as follows. Assume that we perform our blocking algorithms in a social network graph \( G(V, E) \) of \( |V| \) users and \( |E| \) connections. The baseline of classic greedy algorithm has time complexity of \( O(K|E|) \) as we enumerate all users and pick the one with the largest degree, which requires traversing all connections once for each of \( K \) iterations. Here we have \( K \ll |E| \).

In contrast, for our proposed greedy algorithm, most of the time cost lies in the loop of updating the maximum likelihood function \( f \). Consequently, in order to calculate \( f \), we have to go through every user and connection at most once, which has the time complexity of \( O(|V| + |E|) = O(|E|) \). Subsequently, in order to pick out the candidate node \( u \) in each iteration, we need to repeat calculating \( f \) for \( O(|V|) \) times. Combining these factors, we can conclude that the total time complexity for our proposed greedy algorithm is \( O(K|E||V|) \).

Similarly, for the dynamic blocking algorithm, which can be viewed as an online algorithm, we decompose the selection of candidate nodes into \( n \) epochs and in the \( j \)-th \( (j = 1, 2, ..., n) \) epoch, we select \( k_j \) nodes to block, where \( \sum_{j=1}^{n} k_j = K \). In our algorithm, for simplicity, we choose \( k_j = 2^{(-j)} * K \). Then the time complexity in each epoch can be calculated as \( O(k_j|E||V|) \). Finally by combining the time complexity of every epoch together, we obtain that the entire algorithm runs in \( O(K|E||V|) \) time.

Additionally, based on the above analysis, we conclude that the main difference between the time complexity of our proposed algorithm and the baseline greedy algorithm lies in the value of \( |V| \). In order to achieve better performance at relatively low cost, it is better to implement the algorithm in networks with smaller \( |V| \). In reality, when a rumor is generated, we expect to locate it in a short time and conduct the blocking algorithms in a local community that contains all the infected nodes. In that case, the value of \( |V| \) can be lowered to the largest extent by limiting the target network to a local community instead of the entire social network.

**7 EXPERIMENTS**

**Dataset:** We use three datasets to verify the effectiveness of our proposed algorithms. They are extracted from the real world large scale social networks such as Facebook, Twitter and Sina Weibo. Details of the datasets description are listed in Table 1.

**Notations:** Let BlkPer denote the percentage of blocked nodes in all the nodes in the social network. In our simulation, we set BlkPer to three values as 1%, 2% and 10%. To be noticed, in realistic large-scale social networks with millions or even billions of users, 1% may account for up to millions of users and blocking them is not realistic. However, in real world rumor blocking, it is impossible to accomplish any blocking mechanism in a global network scale. It is more reasonable to first locate the rumour source and divide the entire network into communities according to certain attributes of the rumour source (e.g. the geographical information, related friends, etc), and then conduct blocking strategies within a target community with the highest risks. Then, the scale of the target network would be much smaller than the entire network and thus our parameter settings would have more realistic sense.

In all figures, the vertical dashed line is plotted to denote the time when the rumor is detected and our blocking
strategies start working. All the parameters are selected based on empirical results that approximate the realistic scenario. In the experiment, three algorithms are presented for comparison which are listed as follows:

- **Classic Greedy Algorithm**: Greedy algorithm based on descendant order of nodes degree and is used as the baseline algorithm.
- **Proposed Greedy Algorithm**: the order is determined by the maximum likelihood function. By blocking a node, we can generate a new propagation matrix and reach a new maximum survival likelihood value.
- **Dynamic Blocking Algorithm**: This algorithm adjusts to each propagation status, and gradually includes new targeted nodes as long as the cost is within the scope of tolerable user experience.

In Figure 4, we present the simulation results on the Facebook, Twitter and Sina Weibo datasets respectively. Specifically, for each algorithm in each dataset, we repeat the propagation process for 1000 times and take the average value as the general feature. The black curve stands for classic greedy algorithm, blue one for our proposed maximum likelihood greedy algorithm, which blocks all candidates at once and use a metric different from traditional approach, and finally red one for the dynamic blocking algorithm. Obviously, from all the figures, we can see that for all the social network datasets, the rumor infection ratio is decreased to different degrees after the introduction of rumor blockage strategies. As is shown in the results, according to the final infection ratio, the proposed dynamic blocking algorithm performs the best of all the blocking strategies, since the infection ratio (i.e., the number of nodes infected by the rumor) is minimized at the end of propagation under this schema.

From all three datasets, we can see that with the blocking percentage of all the nodes increasing from 1% to 10%, the performances of different blocking strategies vary distinctly. In general, for each algorithm, we can see the trend that the infection ratio tends to be lower with the percentage of nodes being blocked becoming higher. However, for different algorithms, the degrees of that trend are different. Specifically, the dynamic blocking algorithm has relatively the sharpest trend compared to the classic and proposed greedy algorithms. In the Facebook dataset, when the blocking percentage rises from 1% to 2%, the infection ratio barely changes for the classic greedy algorithm. In contrast, the proposed greedy algorithm obtains approximately a 10% improvement in the infection ratio. For the dynamic blocking algorithm, the blocking performance is even better. In addition, when the blocking percentage becomes higher up to 10%, the infection ratios are lowered by approximately 10%, 20% and 30% for the classic greedy, proposed greedy and dynamic blocking algorithm respectively.

One phenomenon needs to be noticed that at the very beginning point of blocking strategies, the proposed maximum likelihood greedy algorithm (i.e. the blue curve) performs slightly better than the dynamic blocking algorithm (i.e. the red curve). Then after a certain amount of iterations, the red curve surpasses the blue curve and stays better than it ever since. The underlying reason is that in the dynamic blocking algorithm, the dynamic property is revealed from the early stage of rumor blocking. In detail, during the initial period of rumor blocking (approximately from 15th to 25th iteration), the blue curve lies under the red one, which indicates the slower propagation rate for the static schema. This can be explained by the fact that the dynamic algorithm does block fewer nodes than the static greedy algorithm at the initial stage of first several iterations. However, after a certain amount of time (about 25 iterations), the dynamic blocking algorithm dominates the proposed greedy algorithm because the dynamic strategy considers the dynamic variation of the topology of the social network in every iteration and constantly introduce new seeds for blocking. In that case, the dynamic blocking algorithm can be regarded as an iterative greedy algorithm.

In the Twitter and Sina Weibo datasets simulation results shown in Figure 4, we can see the similar results to those in the Facebook dataset. The slight difference between them may be caused by the different topologies of the social networks. From the dataset description in Table 1, we know that the Sina Weibo network dataset has much more nodes than the Facebook and Twitter datasets. There are also some differences in average degree, number of connected components and average clustering coefficient, which could influence the rumor propagation process and thus the final infection ratio in the network. For instance, for Sina Weibo dataset with slightly higher average degree, the network has a larger density than the other two. In that case, it is easier for rumor to spread through the network and accelerate the rumor contagion process. This analysis can be verified by the normal rumor propagation curve (i.e. the green curve) especially at the very initial stage of rumor propagation. We can see the slope of the green curve in the Sina Weibo dataset is slightly higher than the other two, which indicates the higher average degree and connected components lead to faster rumor contagion. After the beginning phase, when the infection ratio reaches a certain threshold, the difference between different network topologies becomes minor. On the other hand, when it comes to rumor blocking, the higher average degree and connected components enable more effective blockage due to its higher density.

<table>
<thead>
<tr>
<th>Data Sets</th>
<th>Facebook</th>
<th>Twitter</th>
<th>Sina Weibo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes</td>
<td>4036</td>
<td>81306</td>
<td>1364548</td>
</tr>
<tr>
<td>Number of edges</td>
<td>88234</td>
<td>1768149</td>
<td>31261651</td>
</tr>
<tr>
<td>Average degree</td>
<td>21.85</td>
<td>21.74</td>
<td>22.91</td>
</tr>
<tr>
<td>Number of connected components</td>
<td>36</td>
<td>278</td>
<td>4631</td>
</tr>
<tr>
<td>Average clustering coefficient</td>
<td>0.2731</td>
<td>0.2653</td>
<td>0.3974</td>
</tr>
</tbody>
</table>
Fig. 4. The experimental results of the rumor infection ratio with propagation iterations under different blocking algorithms in the Facebook ((a),(b),(c)), Twitter ((d),(e),(f)) and Sina Weibo ((g),(h),(i)) dataset respectively. The blocking percentage of all the nodes in the social network is set to 1%, 2% and 10% for each dataset.

Fig. 5. Stationary rumor infection ratio under different blocking algorithms with different blocking ratios on the Facebook, Twitter and Sina Weibo datasets respectively. The blocking ratio ranges from 1% to 2% with an interval of 0.1%, which shows the sensitivity of different blocking algorithms.

TABLE 2
False Positive Ratio in Nodes Blocking

<table>
<thead>
<tr>
<th>Data Sets</th>
<th>Facebook</th>
<th>Twitter</th>
<th>Sina Weibo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classic Greedy</td>
<td>42%</td>
<td>40%</td>
<td>70%</td>
</tr>
<tr>
<td>Proposed Greedy</td>
<td>34%</td>
<td>36%</td>
<td>64%</td>
</tr>
<tr>
<td>Dynamic Schema</td>
<td>14%</td>
<td>18%</td>
<td>34%</td>
</tr>
</tbody>
</table>
Fig. 6. Rumor infection ratio under different blocking algorithms with different starting time (iterations of rumor propagation) of blocking on the Facebook, Twitter and Sina Weibo datasets respectively.
In order to further observe the sensitivity of different blocking algorithms over the blocking ratios. We set the blocking ratio range from 1% to 2% with an interval of 0.1%. Thus, we can observe how the final stationary rumor infection ratio varies with the gradual increase of blocking ratio. Figure 5 shows the results of the comparison of the sensitivities of different algorithms. From the figure, we can see that for each blocking ratio, the proposed dynamic blocking schema has the best performance in stationary rumor infection ratio. Furthermore, with the increase of the blocking ratio, our proposed algorithms can decrease the rumor infection ratio at a higher rate than the baseline algorithms, especially at lower blocking ratio. Overall, the experiments demonstrate our proposed algorithms have a better sensitivity in terms of reducing the rumor infection ratio.

To be noticed, in our blocking algorithms, we select a certain amount of nodes from the inactivated ones to minimize the final number of infected nodes. The reason we do this is that the already activated nodes (i.e., the users who have accepted and shared the rumor) have determined status in the network, and we can directly delete the rumor message from their accounts. However, for the inactivated nodes, all of them are at high risk and might cause serious damages. Nevertheless, among all the nodes we choose to block, there might have been nodes that would have not activated in normal rumor propagation without interventions. In other words, it is possible that we commit a "false positive" error in our algorithms. Therefore, we conduct experiments to obtain the statistical "false positive" error rate as an evaluation of our algorithms. We conduct our dynamic blocking schema at 6-th iteration with a blocking rate of 1% for 1000 times and then calculate the statistical average of them. The final results are shown in Table 2. Attributed to the randomness of rumor propagation in each iteration, the results for each iteration fluctuate in a wide range, hence we can only calculate an average for reference. From the table, we see that our algorithms have lower "false positive" error rates than the classic greedy algorithm, and the dynamic schema has the lowest rate. This phenomena demonstrates our maximum likelihood based algorithms are able to select the nodes that are most likely to get activated in normal rumor propagation. The reason we still select nodes that would have not been activated is that these nodes are influential to the entire network and by blocking them, we can minimize the global likelihood of rumor infection. In other words, we sacrifice a small proportion of influential nodes for the benefit of the entire network. In addition, it is also shown that the error rate is lower in small scale networks (e.g. Facebook dataset) than in larger scale ones (e.g. Sina Weibo dataset). The reason is that larger scale networks increase the randomness of rumor propagation, making the prediction of it more difficult.

In our simulation experiments, we assume that the rumor has propagated through the social network for some time according to our proposed model and then at a certain time instant, it is detected by the system. Once the rumor is detected, we start the presented blocking strategies to prevent it from further diffusion. Obviously, in this process, the starting time to block the rumor (i.e. the time instant when we detect the rumor) has enormous impact on the final rumor infection ratio in the entire social network. Thus it is necessary to conduct experiments and analyze to what degree the starting blocking time affect the rumor propagation. Figure 6 shows the experimental results of how the starting blocking time influence the final infection ratio in different algorithms. In all three datasets, we can see that the earlier we start blocking the rumor, the lower the final infection ratio will be. That is easy to comprehend because if we detect the rumor at an early stage, the infection ratio of the entire network would probably be relatively lower. Thus, if we start blocking it, the rumor could be prevented in time and naturally, the final infection ratio will be constrained to a lower level.

In the experiment, we choose three representative values of starting time of blocking as 6, 12, 18, 24, 30 and 36 iterations, which represent the rumor is blocked at different infection stages. Here iterations represent the rumor propagation rounds in the Independent Cascade model. From the experimental results, we can see that if the rumor is blocked at an early stage as in the case of 6 and 12 iterations, our algorithms (both greedy and dynamic blocking algorithms) have distinctly better performances than the classic greedy algorithm. However, if the rumor is blocked in the middle stage as in the case of 18 and 24 iterations, all the blocking algorithms have similarly limited influences on preventing the rumor from further diffusion. Furthermore, in the case of starting blocking the rumor from a late stage such as at the 36th iteration, all of the blocking strategies have almost zero impact on the normal rumor propagation process. The reason is that if the rumor is detected and blocked at a relatively late stage, the rumor has already infected a
large portion of the nodes in the entire network. Thus, it is difficult to restrain the contagion of rumor.

Figure 7 demonstrates the influence of blocking duration on the infection ratio in different datasets. It is generated using the dynamic blocking algorithm and reflects the effect of different block durations on rumor propagation range, i.e., the infection ratio at the end of the propagation. Although in our proposed algorithms, we utilize blocking time as a user experience constraint, we still want to explore the relationship between rumor infection ratio and the blocking duration in order to figure out the bound of blocking time. As is shown in the figure, the longer a node is blocked, the slower the rumor propagates. This benefit, however, is obtained at the expense of declined user experience. The result helps us to analyze the possibility of achieving close performance with less cost. It is also noticeable that this result is coherent to our analysis on User Experience.

From the experimental results, we try to figure out the influence of different rumor blocking algorithms and different blocking durations on the final rumor infection ratio. Though the proposed dynamic algorithm shows the best performance in blocking the rumor to a lower ratio, the computational complexity is also a bottleneck. Thus the exploration of blocking duration may enlighten us on designing a better rumor blocking strategy with less cost.

## 8 Conclusion and future work

In this paper, we investigate the rumor blocking problem in social networks. We propose the dynamic rumor influence minimization with user experience model to formulate the problem. A dynamic rumor diffusion model incorporating both global rumor popularity and individual tendency is presented based on the Ising model. Then we introduce the concept of user experience utility and propose a modified version of utility function to measure the relationship between the utility and blocking time. After that, we use the survival theory to analyze the likelihood of nodes getting activated under the constraint of user experience utility. Greedy algorithm and a dynamic blocking algorithm are proposed to solve the optimization problem based on different nodes selection strategies. Experiments implemented on real world social networks show the efficacy of our method. In our future work, we plan to design more sophisticated rumor blocking algorithms considering the connectivity of the social network topology and node properties. We intend to separate the entire social network into different communities with different user interests and then analyze the rumor propagation characteristics among communities. We are also interested in investigating how to prevent the rumor propagation effectively at a late stage.

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## References


