2D QUATERNION FOURIER TRANSFORM: THE SPECTRUM PROPERTIES AND ITS APPLICATION IN COLOR IMAGE REGISTRATION

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ABSTRACT

We first investigate 2D quaternion Fourier transform (QFT) spectrum relationship between an image and its geometrically transformed counterpart from the aspects of gray images and color images respectively, and then propose a 2D QFT-based color image registration approach, which is able to handle large translation, rotation and scaling. Fourier transform (FT) can be utilized in gray image registration but can not process color images naturally. As the extended version of FT in multidimensional signal processing, QFT is capable of processing three color components of color images together as pure quaternions and is able to deal with color image registration.

1. INTRODUCTION

Image registration is a fundamental task in many fields, such as computer vision, remote sensing, medical imaging and other applications involved in comparison of series of images. Many solutions have been proposed for the recurrent image registration, which include the methods using image correlation functions as well as those using feature points. Phase-correlation function exploiting Fourier transform properties is recently proposed to improve the accuracy of registration. This kind of methods has been extended to the registration of the translated, rotated and scaled gray images [1][2][3].

Hyper-complex Fourier transform based on quaternion algebra becomes a new research branch called quaternion Fourier transform (QFT) [4][5]. QFT can be regarded as an extension of Fourier transform in Hyper-complex domain. A very first application of QFT is stereo matching [6][7]. In this paper, we explore how QFT can deal with color image registration. The QFT spectrum relationship between an image and its geometric transformed counterpart is investigated, respectively from the aspects of gray images and color images. Then we propose a color image registration approach based on 2D QFT that is able to handle large translation, rotation and scaling. This approach processes the three color components of color images together as pure quaternions. In contrast, Fourier transform (FT) has to convert the color image to three one-channel images or monochromatic images, then processes them individually.

2. THE PROPERTIES OF QFT SPECTRUM

2.1. Preliminary of Quaternion Fourier Transform

The concept of quaternion was introduced by Hamilton [8]. It has one real part and three imaginary parts:

$$q = R + iI + jJ + kK , \tag{1}$$

where R, I, J, K are real numbers, and i, j, k are imaginary units. The three operators obey the rules as follows:

$$i^2 = j^2 = k^2 = -1 \text{ and } ij = -ji = k . \tag{2}$$

Given a quaternion $q = R + iI + jJ + kK$, its quaternion conjugate is:

$$q^* = R - iI - jJ - kK , \tag{3}$$

and its modulus can be computed as:

$$|q| = \sqrt{R^2 + I^2 + J^2 + K^2} . \tag{4}$$

Based on quaternion algebra, 2D QFT of signal $f(x,y)$ is defined as follows:

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-iux} f(x,y) e^{-jvy} dxdy . \tag{5}$$

If the signal $f(x,y)$ is real-valued, (5) can be expanded by formulating it as the cascaded FT:

$$\begin{aligned} F(u,v) &= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} f(x,y)\cdot e^{-iux}dx\right)\cdot e^{-jvy}dy \\ &= \int_{-\infty}^{\infty}(\mathbf{R}_1 + i\mathbf{I}_1)\cdot e^{-jvy}dy = \int_{-\infty}^{\infty}\mathbf{R}_1\cdot e^{-jvy}dy + i\int_{-\infty}^{\infty}\mathbf{I}_1\cdot e^{-jvy}dy \\ &= \mathbf{R}_{21} + j\mathbf{I}_{21} + i(\mathbf{R}_{22} + j\mathbf{I}_{22}) = \mathbf{R} + i\mathbf{I} + j\mathbf{J} + k\mathbf{K} \end{aligned} \tag{6}$$

where $\mathbf{R} = \mathbf{R}_{21}, \mathbf{I} = \mathbf{R}_{22}, \mathbf{J} = \mathbf{I}_{21}, \mathbf{K} = \mathbf{I}_{22}$. If $f(x,y)$ has M rows and N columns, $\mathbf{R}(u,v)$, $\mathbf{I}(u,v)$, $\mathbf{J}(u,v)$ and $\mathbf{K}(u,v)$ are $M \times N$ real matrices. As is presented in [4], we totally require $2MN \cdot \log_2 MN$ real number multiplications, which is the same as 2D DFT.

2.2. The QFT Spectrum Properties of Gray Images

Let $f(x,y)$ and $g(x,y)$ be two gray images related by a shift of $(\Delta x, \Delta y)$:

$$f(x+\Delta x, y+\Delta y) = g(x,y). \quad (7)$$

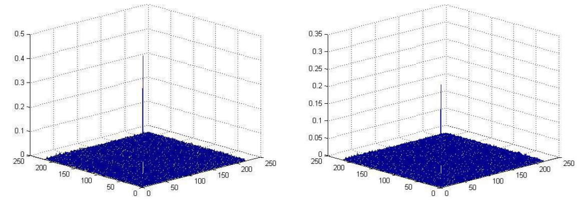
Imposing QFT on both sides of (7), we get:

$$\begin{aligned}&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-iux} f(x+\Delta x, y+\Delta y) e^{-jvy} dxdy \\ &= e^{iu\Delta x}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-iu(x+\Delta x)} f(x+\Delta x, y+\Delta y) e^{-jv(y+\Delta y)} dxdy\right] e^{jv\Delta y}. \\ &= e^{iu\Delta x}\cdot F(u,v)\cdot e^{jv\Delta y} = G(u,v)\end{aligned} \quad (8)$$

The shift between these two images can be presented by:

$$D = [\Delta x, \Delta y]^T = \arg\max\left(F^{-1}\left[\frac{G(u,v)\cdot\alpha(F(u,v))}{|G(u,v)||F(u,v)|}\right]\right), \quad (9)$$

where $\alpha(F(u,v)) = -iF(u,v)i$. Similar to the FT-based phase correlation method, the translation information can be estimated through finding maximum position of the real part in the output via the inverse QFT operation F^{-1}. As shown in Fig.1, QFT phase correlation has one peak, which has the same position as the peak of FT phase correlation.



(a) (b)

Fig.1. FT and QFT phase correlation of two gray images with translation: (a) FT phase correlation, (b) QFT phase correlation.

2.3. 2D QFT Spectrum of Color Images

QFT is the extended version of FT in multidimensional signal processing. Pure quaternion $i\mathbf{I}+j\mathbf{J}+k\mathbf{K}$ can be utilized to represent color images, thus the three color components can be processed together. This is the major reason that quaternions are used for color image processing [9]. Supposing $f_i(x,y), i\in\{1,2,3\}$ are the three color channels of the color image $f(x,y)$, which is respectively represented by $\mathbf{I},\mathbf{J},\mathbf{K}$. Let $[\mathbf{R}_i,\mathbf{I}_i,\mathbf{J}_i,\mathbf{K}_i], i\in\{1,2,3\}$ be the 2D QFT of $f_i(x,y), i\in\{1,2,3\}$ respectively, then quaternion

$$\begin{aligned}\mathbf{q} &= (-\mathbf{I}_1-\mathbf{J}_2+\mathbf{K}_3)+i(\mathbf{R}_1-\mathbf{K}_2-\mathbf{J}_3) \\ &+j(-\mathbf{K}_1+\mathbf{R}_2-\mathbf{I}_3)+k(\mathbf{J}_1+\mathbf{I}_2+\mathbf{R}_3)\end{aligned}, \quad (10)$$

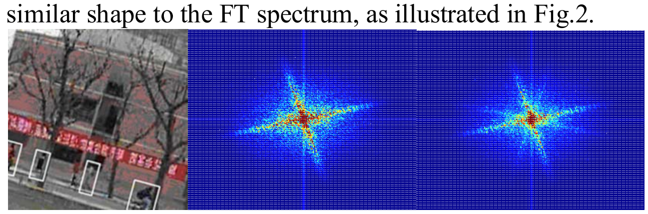
is the representation of 2D QFT of the color image $f(x,y)$. According to (10), the modulus of $\mathbf{q}$ is represented as follows:

$$\begin{aligned}|\mathbf{q}| &= \sqrt{(-\mathbf{I}_1-\mathbf{J}_2+\mathbf{K}_3)^2+(\mathbf{R}_1-\mathbf{K}_2-\mathbf{J}_3)^2+(-\mathbf{K}_1+\mathbf{R}_2-\mathbf{I}_3)^2+(\mathbf{J}_1+\mathbf{I}_2+\mathbf{R}_3)^2} \\ &= \Big((\mathbf{R}_1-\mathbf{K}_2)^2+(\mathbf{I}_1+\mathbf{J}_2)^2+(\mathbf{R}_2-\mathbf{K}_1)^2+(\mathbf{I}_2+\mathbf{J}_2)^2+2\mathbf{R}_3(\mathbf{I}_2+\mathbf{J}_1) \\ &-2\mathbf{K}_3(\mathbf{I}_1+\mathbf{J}_2)-2\mathbf{I}_3(\mathbf{R}_2-\mathbf{K}_1)-2\mathbf{J}_3(\mathbf{R}_1-\mathbf{K}_2)+\mathbf{R}_3^2+\mathbf{I}_3^2+\mathbf{J}_3^2+\mathbf{K}_3^2\Big)^{\frac{1}{2}}\end{aligned} \quad .(11)$$

As a special case that the three color channels are the same, we take all components $\mathbf{R}_i, i\in\{1,2,3\}$ as $\mathbf{R}_s$ and $\mathbf{I}_i,\mathbf{J}_i,\mathbf{K}_i$ as $\mathbf{I}_s,\mathbf{J}_s,\mathbf{K}_s$, therefore the equation (11) is reformed as:

$$|\mathbf{q}| = 2.5[(\mathbf{R}_s-\mathbf{K}_s)^2+(\mathbf{I}_s+\mathbf{J}_s)^2]+0.5[(\mathbf{R}_s+\mathbf{K}_s)^2+(\mathbf{I}_s-\mathbf{J}_s)^2]. \quad (12)$$

The first bracket in (12) dominates the shape of the QFT magnitude spectrum, which has the same formation as the FT magnitude spectrum of one-channel gray image as discussed in section 3. When the energy distribution of the three components in a color image is varied smoothly, it is observed that the QFT spectrum of the color image has similar shape to the FT spectrum, as illustrated in Fig.2.



(a) (b) (c)

Fig.2. Illustration of The FT and QFT spectrum of a color image: (a) Original color image, (b) The magnitude of FT spectrum of one color channel of the color image, (c) The modulus of QFT spectrum of the color image.

3. COLOR IMAGE REGISTRATION METHOD

Assuming real-valued image $g(x,y)$ is a translated, rotated and scaled replica of $f(x,y)$, that is:

$$\begin{aligned}g(x,y) = f\big(&s[(x-x_0)\cos\theta_0+(y-y_0)\sin\theta_0]+\Delta x, \\ &s[-(x-x_0)\sin\theta_0+(y-y_0)\cos\theta_0]+\Delta y\big)\end{aligned}, \quad (13)$$

where θ_0, (x_0,y_0) and s are the rotation angle, the center of rotation and the scaling factor respectively. We can reform (13) as:

$$\begin{cases} g(x,y) = f\big(s(x\cos\theta_0+y\sin\theta_0)+\Delta x' \\ \quad ,s(-x\sin\theta_0+y\cos\theta_0)+\Delta y'\big) \\ \Delta x' = -s(x_0\cos\theta_0+y_0\sin\theta_0)+\Delta x \\ \Delta y' = s(x_0\sin\theta_0-y_0\cos\theta_0)+\Delta y \end{cases}. \quad (14)$$

It can be deduced from (14) that the QFT magnitude is invariant to translation $(\Delta x', \Delta y')$ but retains the effects of rotation and scaling as follows:

$$\|G(u,v)\| = s^{-2}\left\|F\left(s^{-1}(u\cos\theta_0+v\sin\theta_0), s^{-1}(-u\sin\theta_0+v\cos\theta_0)\right)\right\|. \quad (15)$$

If we formulate $G(u,v)$ as a quaternion, i.e. $\mathbf{R}+i\mathbf{I}+j\mathbf{J}+k\mathbf{K}$, its modulus consists of two complimentary parts:

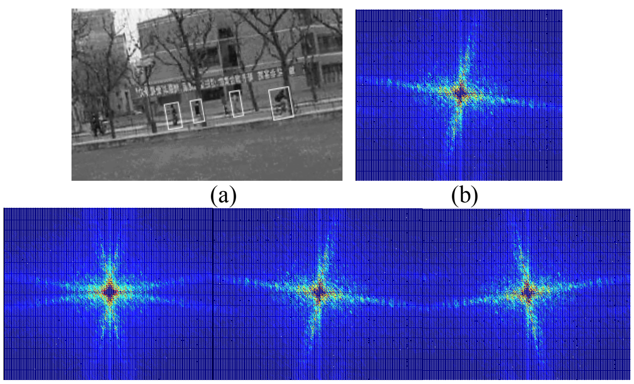
$$\|G(u,v)\|_1 = \sqrt{(\mathbf{R}-\mathbf{K})^2+(\mathbf{I}+\mathbf{J})^2}, \quad (16)$$

and

$$\|G(u,v)\|_2 = \sqrt{(\mathbf{R}+\mathbf{K})^2+(\mathbf{I}-\mathbf{J})^2}. \quad (17)$$

These two complementary components have the same geometric transformation with regard to rotation and scaling.

The FT and QFT spectrum of an image are shown in Fig.3, where subfigures (a) and (b) are respectively the original image and the magnitude of FT. Compared with the FT spectrum, the QFT spectrum contains two complementary components which have the same geometric transformation as displayed in Fig.3(c). If we change QFT to 2D-FT, that is the complex unit i substitutes j in the transform kernel of (6), we can find that $\mathbf{R}-\mathbf{K}$ and $\mathbf{I}+\mathbf{J}$ are the real part and the imaginary part of 2D-FT, respectively. The magnitude of $(\mathbf{R}-\mathbf{K})+i(\mathbf{I}+\mathbf{J})$ is shown in Fig.3(d), which is one complementary component having the same spectrum as the one shown in Fig.3(b). For a conjugate quaternion pair have the same modulus, the magnitude of $(\mathbf{R}+\mathbf{K})+i(\mathbf{I}-\mathbf{J})$ is the other complementary component as shown in Fig.3(e). Both of the components contain the information of geometric transformation and can be utilized to estimate translation, rotation and scaling parameters.



(a) (b) (c) (d) (e)

Fig.3. Illustration of the FT and QFT spectrum: (a) Original image, (b) The magnitude of FT spectrum, (c) The magnitude of QFT spectrum, (d) The magnitude of one component formulated as $(\mathbf{R}-\mathbf{K})+i(\mathbf{I}+\mathbf{J})$, (e) The magnitude of the other component formulated as $(\mathbf{R}+\mathbf{K})+i(\mathbf{I}-\mathbf{J})$.

In Pseudo-Log-Polar representation, (15) is formed as:

$$\hat{G}_i(\log\rho,\theta)=s^{-2}\hat{F}_i(\log\rho-\log s,\theta+\theta_0),i\in\{1,2\}. \quad (18)$$

The term $\hat{G}_i(\rho,\theta)$ is the equivalent version of $\|G(u,v)\|_i$ in polar coordinates. Let $\log\rho=\phi$ and $-\log s=\phi_0$, then (18) is reformed as:

$$\hat{G}_i(\phi,\theta)=s^{-2}\hat{F}_i(\phi+\phi_0,\theta+\theta_0),i\in\{1,2\}. \quad (19)$$

Ignoring the effects of scale s^{-2}, it is noted that the registration problem is able to be first formulated as the estimation of the translation parameter (ϕ_0,θ_0).

Replacing $F(u,v),G(u,v)$ in (9) by 2D QFT of a color image pair, we can estimate the translation information between two color images. As displayed in Fig.4, translation parameters are estimated by locating the peak location in the real part of the output from (9).

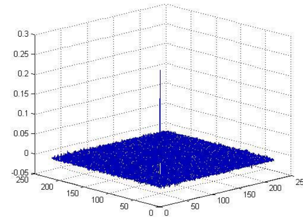


Fig.4. QFT phase correlation of two color images with translation

Similarly, substituting 2D QFT of color images f and g for F,G in (15), the magnitudes computed by (11) are utilized to estimate rotation and scaling for original color images. The 2D QFT-based color image registration algorithm can be summarized by Table 1.

Table 1 The 2D QFT-based color image registration algorithm

A.	Impose 2D QFT on the input color image pair $f(x,y)$ and $g(x,y)$ according to the above description and the outputs are formed as (15)
B.	Calculate quaternion magnitude of the outputs as shown in (11)
C.	Implement Pseudo-Log-Polar transformation on two magnitude spectrums acquired in step B
D.	Estimate the log scale and rotation parameter (ϕ_0,θ_0) or $(-\log s,\theta_0)$ as shown in (19) between two magnitude spectrums based on QFT phase correlation
E.	Taking the image center as the rotation center, warp the reference color image $f(x,y)$ by the estimated scale and rotation parameters (s,θ_0) to $f'(x,y)$
F.	Estimate the translation parameters $(\Delta x,\Delta y)$ between the warped color image $f'(x,y)$ and the transformed image $g(x,y)$ based on QFT, thus the geometric transformation parameters $(s,\theta_0,\Delta x,\Delta y)$ are all acquired

4、 EXPERIMENTAL RESULTS

We now present the experimental results of geometric parameters estimation of four color image pairs. Fig.5(a)-(d) are original color image pairs (the left one of the pair is the reference image and the right one is the transformed image) and (e)-(h) are middle and final warped results when estimating transformation parameters of (a)-(d), respectively. Taking Fig.5(a) and (e) for example, the left image of Fig.5(e) is $f'(x,y)$ in Table 1, warped from the reference image by estimated rotation and scaling parameters, and the right one is produced by translating $f'(x,y)$ with estimated translation parameters between $f'(x,y)$ and the transformed image namely the right one of pair (a). Through the comparison of the final warped results and the transformed images, it can be confirmed that the algorithm estimates the geometric parameters reliably. Rotation, scaling and translation estimation results of the four color image pairs are show in Table 2, where the ground truth of (a) is from [3] and the other pairs are transformed manually according to the ground truth.

5、 CONCLUSION

We have investigated the QFT spectrum relationship between an image and its translated, rotated and scaled replica, respectively from the aspects of gray images and color images. Our method is no better than the FT-based method on gray image registration. But the biggest advantage of our algorithm is that it can process color images directly, without losing color information. Besides color image registration task, other potential applications of Hyper-complex Fourier transform are of most interest in our future work.

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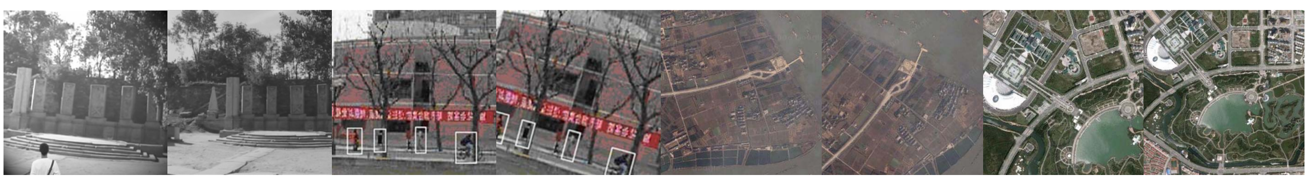
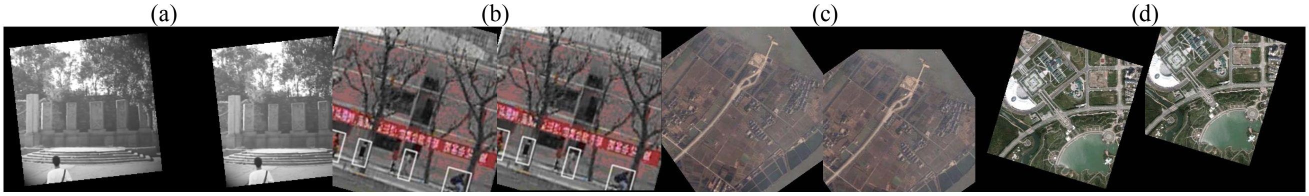


(a) (b) (c) (d)



(e) (f) (g) (h)

Fig.5. The experimental results for testifying the proposed registration method
(a)-(d): Original image pairs (252pixel×252pixel; 237pixel×237pixel; 357pixel×351pixel; 349pixel×339pixel).
(e)-(h): Middle and final warped results.

TABLE 2 Rotation, Scaling and Translation Estimation Results

Image Pairs	Ground Truth (angle, scale)	Estimated (angle, scale)	Estimated (△x, △y)
(a)	(5.625°, 0.843)	(6.3281°, 0.8421)	(2, 57)
(b)	(-15°, 1.15)	(-15.4687°, 1.1510)	(-3, -1)
(c)	(35°, 1.05)	(35.1563°, 1.0603)	(49, -22)
(d)	(-17°, 0.78)	(-16.8750°, 0.7758)	(-48, -35)