Image Denoising Using Local Tangent Space Alignment

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Outline

1. Introduction

2. Our method and Results
   - Local tangent space alignment (LTSA) method
   - Image denoising using LTSA
   - Experiment results

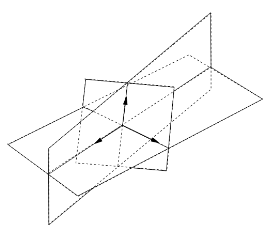
3. Conclusion and future work

THE PROBLEM AND NOWADAYS SOLUTION

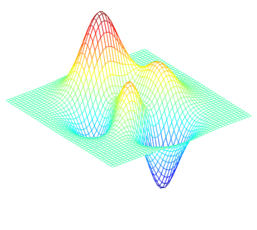
- Problem: image denoising.
- Importance: test bench for models and techniques.
- One state of the art algorithm: Denoising using KSVD.
- KSVD assume linear representation for image while low dimension manifold representation is more likely to be true.

MOTIVATION OF OUR ALGORITHM

From linear to non-linear: sparse representation to manifold



(a) Planes spanned by pairs of vectors



(b) A manifold

$$\min_{X} \quad \lambda \|X - Y\|_F^2 + F(X) \tag{1}$$

WHAT IS LTSA

- It is a manifold learning method.
- Data set: $z_i \in \mathbb{R}^D$, $i = 1, 2, \cdots, N$
- It has two steps:
  1. Find A_i, where $Z_i \in \mathbb{R}^{D\times(k+1)}$, $\overline{P_k} = I_{k+1} - \frac{1}{k+1}\mathbf{1}\mathbf{1}^T$ and $L_i \in \mathbb{R}^{D\times d}$ is orthogonal.

$$\min_{L_i, A_i} \quad \|Z_i \overline{P_k} - L_i A_i\|_F^2. \tag{2}$$

  2. Find Θ using A_i's, where θ_i is the low demension coordinate of z_i on the manifold.

$$\min_{,\{T_i\}} \quad \sum_{i=1}^{N} \|\Theta_i \overline{P_k} - T_i A_i\|_F^2. \tag{3}$$

APPLY LTSA TO IMAGE DENOSING

- Replace Θ_i in (3) by $\widetilde{X}S_i$, where columns of $\widetilde{X}$ are vectorized image patches and S_i is the selection Matrix.
- We define the new regularity function $F(x)$ to be

$$\sum_{i=1}^{N} \|\widetilde{X}S_i(\overline{P_k} - V_i^T V_i)\|_F^2, \tag{4}$$

where V_i is made by the right singular vectors of matrix $Z_i\overline{P_k}$.

- The new objective function becomes

$$\min_X \lambda\|X - Y\|_F^2 + \sum_{i=1}^{n} \|\widetilde{X}S_i(\overline{P_k} - V_i^T V_i)\|_F^2, \tag{5}$$

which has a closed form solution.

TWO MODIFICATION ON LTSA

1. Pre-denoising: Using Local Linear Projection(LLP) first and derive more accurate value of V_i.
2. Local dimension: Experiments show the dimension of the manifold should be variable at different area, e.g. flat patches lie in a lower dimension local space.

ALGORITHM FLOWCHART

Task: Denoising the given image Y knowing the noise power σ.

1. ***Basic denoising stage with d_i derived:***
   - Vectorize the image patch of Y to be z_i.
   - For each z_i, find k nearest neighbors and form Z_i.
   - Decompose $Z_i\overline{P_k}$ using the singular value decomposition (SVD), $Z_i\overline{P_k} = U \quad V$. The singular values are $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_D \geq 0$.
   - Set $d_i = min_d\{d| \sum_{j=d}^{D} \lambda_j^2 < \alpha(k+1)D\sigma^2\}$.
   - Set $d_i' = min_d\{d| \sum_{j=d}^{D} \lambda_j^2 < \beta(k+1)D\sigma^2\}$.
   - Project $Z_i\overline{P_k}$ onto the first d_i' eigenvectors and see the projections as the estimates of the corresponding image patches.
   - Average the estimates at each pixel to get X'.

2. ***Manifold-based denoising stage:***
   - Vectorize the image patch of X' to be z_i.
   - For each z_i, find k nearest neighbors and form Z_i.
   - Decompose $Z_i\overline{P_k}$ using singular value decomposition (SVD) $Z_i\overline{P_k} = U \quad V$. The singular values are $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_D \geq 0$.
   - Derive V_i to be the upper d_i rows of V.
   - Set $err_i = \sum_{j=d_i+1}^{D} \lambda_j^2$.
   - Set $\lambda = \gamma \frac{\text{mean}(err_i)}{(k+1)D\sigma^2}$.
   - Apply $\hat{X} = (\lambda I + \sum_{i=1}^{N} \sum_{j=1}^{k+1} ({A_i^j}^T A_i^j))^{-1} \lambda Y$.

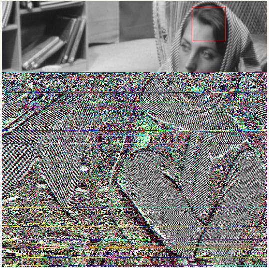
PSNR COMPARE

Table: Summary of the denoising PSNR results in decibels. In each cell, two denoising results are reported and the higher one is in the bold face. Left: Results of KSVD. Right: Results of the method proposed in this paper. We set $k = 100$, $\alpha = 0.5$, $\beta = 0.8$, $\gamma = 7700$. The factors are chosen from experiments.

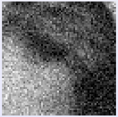
σ/PSNR	Lena		Barb		Boats		House		Peppers	
2/42.14	42.62	42.62	42.84	**43.00**	42.60	42.60	45.14	**45.20**	**45.32**	45.13
5/34.12	38.42	**38.45**	37.17	**37.47**	36.15	**36.20**	39.19	**39.95**	41.94	**42.83**
10/28.08	35.59	**35.75**	34.42	**34.77**	**31.72**	31.64	38.43	**38.86**	33.61	**34.07**
15/24.58	36.11	**36.53**	32.62	**32.81**	31.64	**31.81**	**39.37**	39.27	33.89	**34.51**
20/22.06	30.83	**31.48**	29.90	**30.58**	29.81	**30.07**	**33.82**	33.69	35.22	**35.27**
25/20.19	30.51	**30.82**	30.73	**31.70**	28.76	**29.45**	30.53	**30.84**	31.28	**31.65**

VISUAL QUALITY COMPARE

Original Image

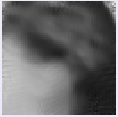


Noisy Image (20.17dB)



Denoised Image (KSVD,30.73dB)

Denoised Image (our method,31.70dB)



CONCLUSION AND FUTURE WORK

- Our research has led to a novel manifold-based image denoising method, leading to state-of-the-art performance and surpassing the the KSVD denoising method.
- The results in the paper do not reflect the full potential of the proposed approach, and we have a lot to do in the future, e.g., reducing the computation complexity so that it can be used for large images. More analysis of the image properties can make the performance even better.

Q & A

Thank You!