New Bounds on Image Denoising: Viewpoint of Sparse Representation and Non-Local Averaging

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Outline

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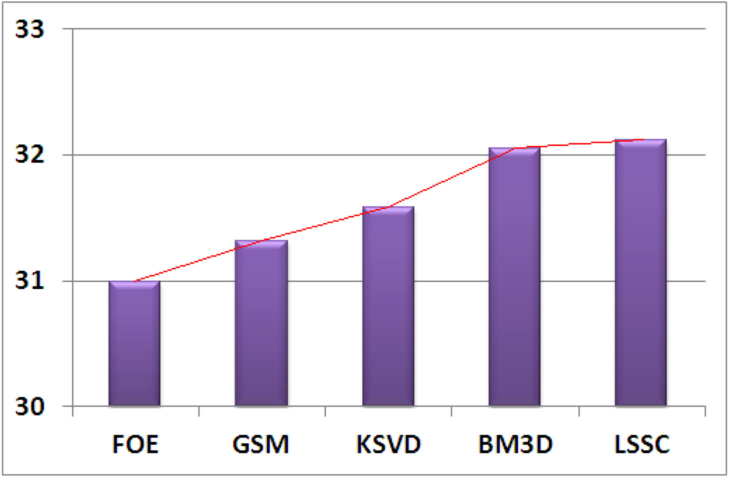
1. Motivation and goal
2. Prior art vs requirements
3. New denoising bounds
4. Experiment results
5. Conclusion

Is Denoising Dead



Have we reach the bound?

Patch Based Denoising Problem

An image I is divided into N patches $\mathbf{z}_i \in \mathbb{R}^n$.

$$\mathbf{y}_i = \mathbf{z}_i + \eta_i \tag{1}$$

For a denoising algorithm f,

$$MSE(f) = \frac{1}{nN} \sum_i \mathbb{E} \|\hat{\mathbf{z}}_i(f) - \mathbf{z}_i\|_2^2. \tag{2}$$

Goal

Ideal bound:

$$\min_{f \in \mathcal{F}} MSE(f) \tag{3}$$

Requirement of the new bound:

1. Establish $\mathcal{F}$: Nowadays algorithms can be well approximated with f's in it.
2. Derive the bound: No additional assumption on the image model.

Outline

Bayasian bound (Levin et.al CVPR 2011)

A general prior $p(\mathbf{z})$: from a large image dataset.

$$\hat{\mathbf{z}}_i = \int_{\mathbf{z}} p(\mathbf{y}_i|\mathbf{z})p(\mathbf{z})dz \tag{4}$$

Conflict with requirement 1: Nowadays algorithms already use adaptive prior, validated to be more efficient than a general one.

Affine bias model based bound(Chatterjee et.al TIP 2010)

$$E\left[\|\mathbf{z}_i - \hat{\mathbf{z}}_i\|^2\right] \geq \mathrm{Tr}\left\{\left(\mathbf{J}_i + \mathrm{cov}_k^{-1}(\mathbf{z})\right)^{-1}\right\} \tag{5}$$

Conflict with requirement 2: while calculating $\mathbf{J}_i$, they made an assumption on image patches

$$\frac{1}{N_i}\sum \mathbf{z}_j = \mathbf{z}_i, \tag{6}$$

where

$$\|\mathbf{z}_j - \mathbf{z}_i\|^2 \leq \gamma. \tag{7}$$

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Techniques for denoising

Requirement 1: Establish $\mathcal{F}$: Nowadays algorithms can be well approximated with f's in it.

Main techniques shared in nowadays algorithms:

Technique	Image Property
Sparse representation	Sparsity
Non-local averaging	Non-local Similarity

Our framework

Denoising process under parameter set $(K, \{N_i, \mathbf{U}_{k,i}\})$:

1. Clustering geometric similar patches into K groups, Locally Adaptive Regression Kernels as feature.
2. Apply PCA to each group to obtain $\mathbf{U}_k$, $\mathbf{U}_{k,i}$ contains a few columns of $\mathbf{U}_k$.
3. Find N_i radiometric similar patches of $\mathbf{z}_i$.
4. Wiener filtering:

$$\hat{\mathbf{z}}_i = \mathbf{u}_i + \mathrm{Cov}_i(\mathrm{Cov}_i + \sigma^2 I)^{-1}(\mathbf{y}_i - \mathbf{u}_i), \tag{8}$$

where

$$\mathbf{u}_i = \frac{1}{N_i}\sum_j \mathbf{U}_{k,i}\mathbf{U}_{k,i}^T\mathbf{y}_j \tag{9}$$

and

$$\mathrm{Cov}_i = \mathbf{U}_{k,i}[\mathrm{Cov}(\{\mathbf{U}_{k,i}^T\mathbf{y}_j\}) - \sigma^2 I]_+\mathbf{U}_{k,i}^T. \tag{10}$$

New bound vs. Requirement

Denote $\mathcal{F}$ as all the previous parameterized algorithms.

Nowadays algorithm is approximated by a certain f using parameter set $(K, \{N_i, \mathbf{U}_{k,i}\})$. (*Requirement 1 satisfied*)

We find $\min_{f \in \mathcal{F}} MSE(f)$ by enumerating $(K, \{N_i, \mathbf{U}_{k,i}\})$. (*Requirement 2 satisfied*)

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Parameters

The patch size is 11×11, so $n = 121$.

Parameter set search range:

1. $K \in \{5, 10, 15, 20, 25\}$.
2. $N_i \in [1, 20]$.
3. $\mathbf{U}_{k,i}$ could use any set of columns in $\mathbf{U}_k$.

Images for testing

parrot

peppers

house

cameraman

New bounds result

Table 1: MSE results under different images

	parrot	peppers	house	cameraman
BM3D	47.37	34.91	20.83	41.84
Bound	37.89	31.03	14.94	36.02
Bound*	38.51	31.50	16.56	36.68

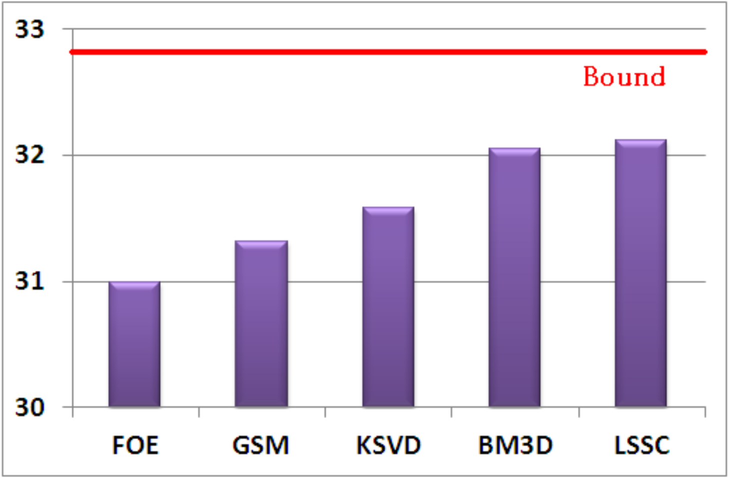
$\sigma = 15$

Bound: find similar patches using clean patches.

Bound*: find similar patches using denoised patches of BM3D.

New bounds result

Averaged SNR gaps between BM3D and Bound: **0.9dB**.



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Conclusion

We proposed a new bound in the viewpoint of sparse representation and non-local averaging.

Experiments results show: there still leaves room for improvement.

Finding image property other than sparsity and non-local similarity is encouraged.